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"Traffic modelling and simulation"

Syllabus

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UNIVERSITÄT KLAGENFURT

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

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Course Goals and general issues/aspects covered in transportation

(Course Goals: Learning Objectives and Learning Outcomes)

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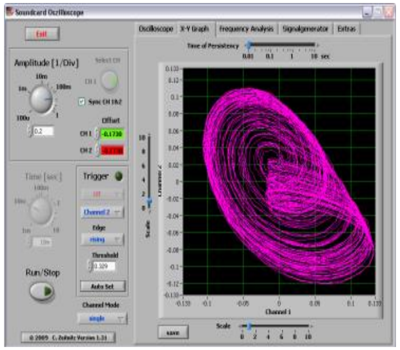
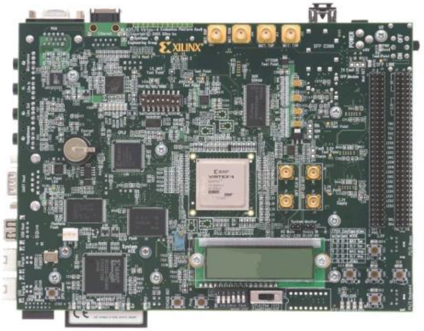
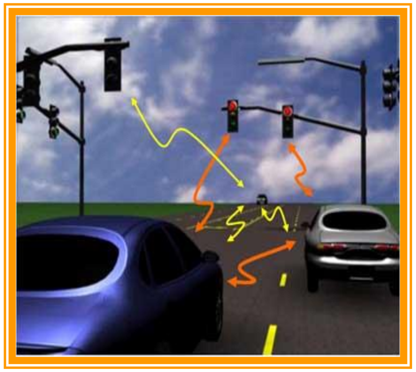
- **Objectives of the Lecture/course**
  - ✓ Learning how to represent systems in transportation (e.g. mathematical models and graphical models)
  - ✓ Learning how to analyse systems in transportation (e.g. use of MATLAB and Neural Networks for simulation)
  - ✓ Understanding the techniques of Control/Optimization
  - ✓ Understanding the techniques of Forecasting
- **Mastering of Modeling (Concepts/Techniques)**
  - ✓ Differential equations (ODE, PDE) in transportation
  - ✓ Neural Networks & Graphs theory in transportation
  - ✓ Oscillatory theory in transportation (see Kuramoto system)
- **Mastering of Simulation (Concepts /Techniques)**
  - ✓ Traffic simulation (microscopic macroscopic mesoscopic & nanoscopic level of details)
  - ✓ Neurocomputing: The main focus is on both ANN & RNN.
- **Development of Traffic Sensors (Models)**
  - ✓ *Machine vision based dynamic systems* (e.g., use of ODEs & PDEs for traffic Sim., use of ODEs & PDEs for Image processing, Use of Coupled oscillators for traffic control)
- **Control & Optimization in Transportation**
  - ✓ Adaptive Traffic Control
  - ✓ Dynamic Vehicle Routing
  - ✓ Dynamic Train Scheduling







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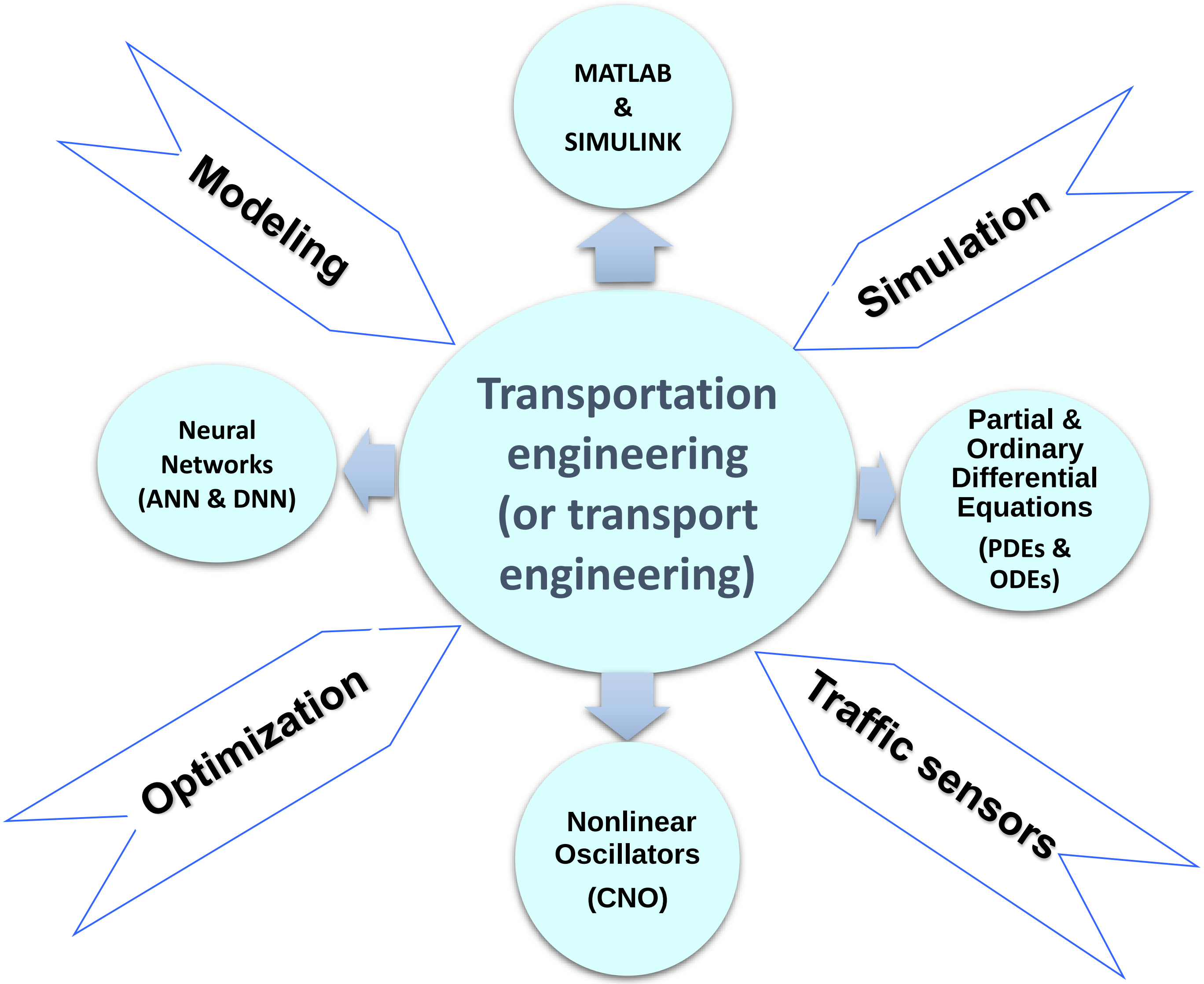
Basic Instruments/Concepts used in the Course/Lecture


MATLAB
&
SIMULINK
Modeling
Simulation
Neural
Networks
(ANN & DNN)
Transportation
engineering
(or transport
engineering)
Partial &
Ordinary
Differential
Equations
(PDEs &
ODEs)
Optimization
Traffic sensors
Nonlinear
Oscillators
(CNO)

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Important scientific fronts covered by the Course/Lecture

Scientific fronts addressed in the Course

Topic	Tools
Fundamentals of Optimization	• MATLAB
Basics of Neural Networks & Simulation Algoritms	• MATLAB
Traffic Modeling and Simulation	• MATLAB • Neural Networks
Traffic Control/Optimization	• MATLAB • Neural Networks
Supply Chain Networks & Logistics	• MATLAB • Neural Networks

Important aspects to be covered in this Course/Lecture by the Lecturer

Important aspects to be investigated in each chapter

- **Chapter 1. Introduction to traffic modeling, simulation, management and control**
  - ✓ Description of selected illustrative examples of Road traffic, Rail traffic, Air traffic, Boat traffic, and Supply Chain Networks (traffic of Goods)
  - ✓ Discussing the pros and cons of modelling and simulation in transportation
  - ✓ Discussing the key challenges related to traffic modeling and simulation
  - ✓ Classification of traffic models: Static models; Dynamic models; Continuous models; Discrete models; Deterministic-, Stochastic-, and Event-based models.
  - ✓ Physical Interpretation of the modeling procedure: White-box modeling; Black-box modeling; Grey-box modeling.

- **Chapter 2. Overview on traffic models and traffic simulation tools**
  - ✓ Presentation of selected models of specific road and rail traffic scenarios
  - ✓ Definition of key/basic terminologies in road traffic and rail traffic
  - ✓ Presentation of some concrete illustrative examples of the afore-mentioned basic terminologies both in road traffic and rail traffic
  - ✓ Description of some commonly used traffic simulation tools for: Macroscopic-microscopic-, mesoscopic-, nanoscopic- traffics, and supply chain networks.

Important aspects to be investigated in each chapter

Chapter 3. Basics of neural networks and application in transportaion

- ✓ Introduction to neural networks: Definition; History; Objectives and advantages of the use of neural networks; Selected applications of Neural Net. in Transportation.
- ✓ Neural networks structures: Biological Neurons (BN) Vs. Artificial Neurons (AN); How to construct BN-Networks (BNNs) & AN-Networks (ANNs)?; Functioning principle of BNN & ANN; Importance of analysing convergence properties of ANNs.
- ✓ Learning/Training phase of artificial neural networks (ANNs): Description of some commonly used learning strategies/rules; Illustrative examples of how the rules are used in/for specific applications
- ✓ Usage phase of ANNs: Illustration through case studies; Convergence properties.
- ✓ Neuron model: Single-input neuron; Transfer functions; Multiple input neuron.
- ✓ Network architecture: Single layer of neurons; Multiple layers of neurons; Recurrent neural networks (RNNs); Advantages of RNNs compared to ANNs for Traffic Sim.
- ✓ Analytical investigation of the convergence properties of neural network platform (or architecture). Derivation of the analytical conditions to ensure the convergence.
- ✓ Sample application examples of how to solve problems using neural networks
- ✓ Some exercises to check the knowledge acquired in the chapter

Important aspects to be investigated in each chapter

❑ Chapter 4. Basics of optimization and simulation algorithms/tools for optimization

- ✓ General introduction to optimization
- ✓ MATLAB Toolboxes for optimization
- ✓ Linear programming Toolbox (LP)
- ✓ Quadratic programmin Toolbox (QP)
- ✓ Case studies: Some illustrative application examples
- ✓ Neural Nets. (networks) theory for optimization: ANN; Gradients method; RNN
- ✓ Case studies: Solving concrete selected optimization problems using Neural Nets.

❑ Chapter 5. Optimization in graph networks with applications in transportation

- ✓ Mathematical modeling of the Shortest path problem (SPP) in graph networks with applications in transportation.
- ✓ Mathematical modeling of the Minimum spanning tree (MST) in graph networks with applications in transportation.
- ✓ Mathematical modeling of the Traveling Salesman Problem (TSP) in graph networks with applications in transportation.

Important aspects to be investigated in each chapter

- Chapter 6. Optimization/Control in road transportation
  - ✓ Mathematical modeling and control of the dynamics of traffic flow on Highway (**<u>As a didactic example</u>:** The complete teaching of this subject/topic is proposed below ---> see the last part of this talk entitled **"Didactic example“**).
  - ✓ Mathematical modeling of the traffic signal control principle at isolated (local) traffic junction: Traffic signals splitting, Green signal sharing, etc.
  - ✓ Mathematical modeling of the traffic signal control principle in a network of coupled traffic junctions: Traffic signals splitting, Green signal sharing, etc.

- Chapter 7. Optimization/Control in railway transportation
  - ✓ Mathematical modelling of the train dynamics and optimization of the Energy consumption
  - ✓ Mathematical modeling and optimization of the railway blocking problem
  - ✓ Optimization of the train trajectory: Modelling concept and simulation algorithm/method
  - ✓ <u>Case study 1</u>: Derivation of the “Optimization model” for two-train trajectory
  - ✓ <u>Case study 2</u>: Trains on time - Optimization and Scheduling of railway timetables.

Important aspects to be investigated in each chapter

- **Chapter 8. Supply chain networks (SCN): Modelling & Analysis of the Dynamics of (SCN)**
  - ✓ Overview of supply chain networks
  - ✓ Modelling principle of supply chain networks
  - ✓ Modelling and optimization of the assignment problem in a supply chain network
  - ✓ Supply chain management optimization problem

- **Chapter 9. Scheduling: Fundamentals of Scheduling and Applications in Transportation**
  - ✓ Overview of scheduling
  - ✓ Principles of scheduling
  - ✓ Modelling job-shop scheduling problems
  - ✓ Railway scheduling by network optimization problem
  - ✓ Modeling of the railway scheduling problem and time-tables optimization
  - ✓ Modeling and optimization of the Crew scheduling problem in Railway transportation

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Selected results So far published by some of our Master and PhD students who have attended this Course/Lecture.

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(Sample Result 1. Published [1]) Modelling of Traffic Flow

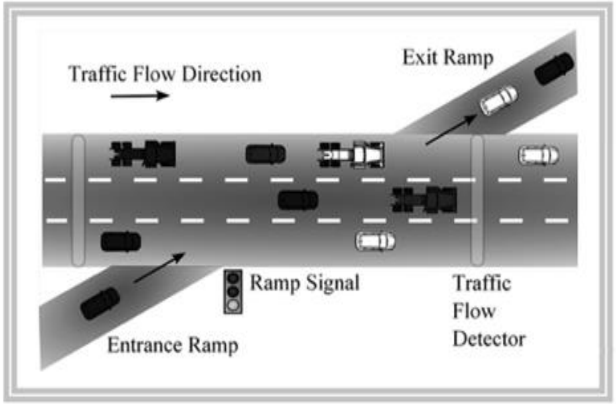

Fig 1. General representation of the traffic flow on a road segment of finite length. This representation shows three lanes with overtaking possibility. Both on-ramp and off-ramp are also illustrated [6].

$$Section\ S_1:\begin{cases} Input\ 1: q_{in} = [q_0 + r(t)] \\ Output\ 1: q_{out} = [\alpha\rho_1 v_1 + (1-\alpha)\rho_2 v_2] \end{cases} \quad (1)$$

$$Section\ S_2:\begin{cases} Input\ 2: q_{in} = [\alpha\rho_1 v_1 + (1-\alpha)\rho_2 v_2] \\ Output\ 2: q_{out} = [\alpha\rho_2 v_2 + (1-\alpha)\rho_3 v_3 + s] \end{cases} \quad (2)$$

$$Section\ S_3:\begin{cases} Input\ 3: q_{in} = [\alpha\rho_2 v_2 + (1-\alpha)\rho_3 v_3] \\ Output\ 3: q_{out} = [\alpha\rho_3 v_3 + (1-\alpha)\rho_3^* v_3^*] \end{cases} \quad (3)$$

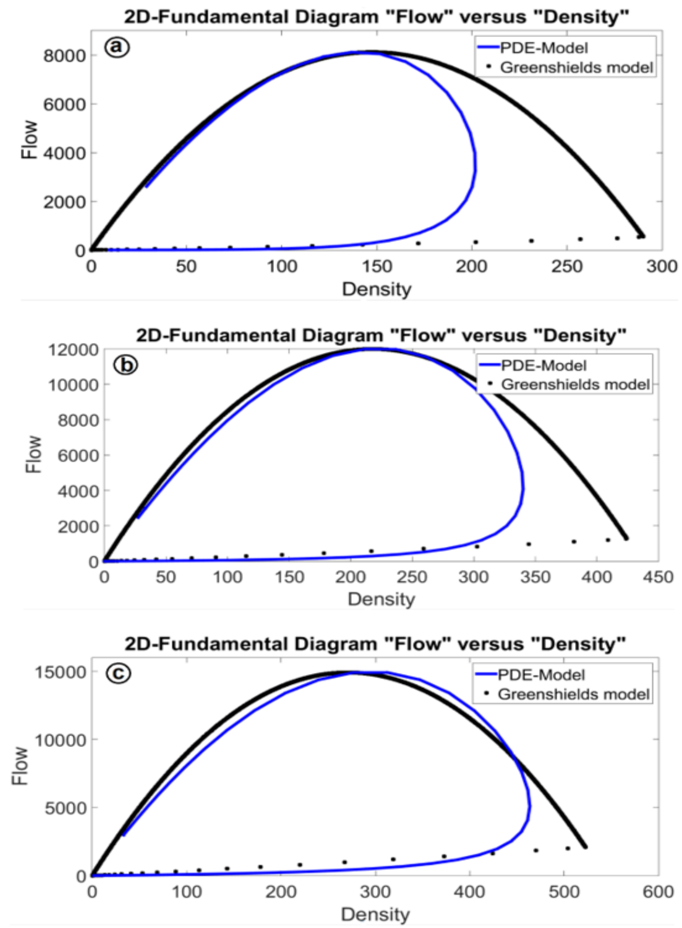

Fig. 2. Fundamental diagram expressing the evolution of "flow" versus "density" in the three sections of the road segment in figure 3. The plots in (a), (b), and (c) correspond to sections 1, 2, and 3, respectively.

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(Sample Result 2. Published [2])

Simulation of Traffic Flow

PDE model for traffic flow

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial Q}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} = a[V_e(\rho) - v] + c_0 \dfrac{\partial v}{\partial x} \end{cases}$$

$\boldsymbol{\rho}$: Traffic density;

$\boldsymbol{v}$: Traffic speed;

$\boldsymbol{Q} = \boldsymbol{\rho v}$: Traffic flow

$\boldsymbol{c_o}$: Propagation speed of disturbance (e.g. shockwave)

$\boldsymbol{a}$: Sensitivity term;

$\boldsymbol{v_f}$: free flow speed

$\boldsymbol{\rho_m}$: max density

$$\boldsymbol{Q} = \boldsymbol{\rho v}$$

$$V_e(\rho) = v_f\left\{\left[1 + exp\left(\frac{\rho * \rho_j^{-1} - 0.25}{0.06}\right)\right]^{-1} - 3.92 * 10^{-6}\right\}$$

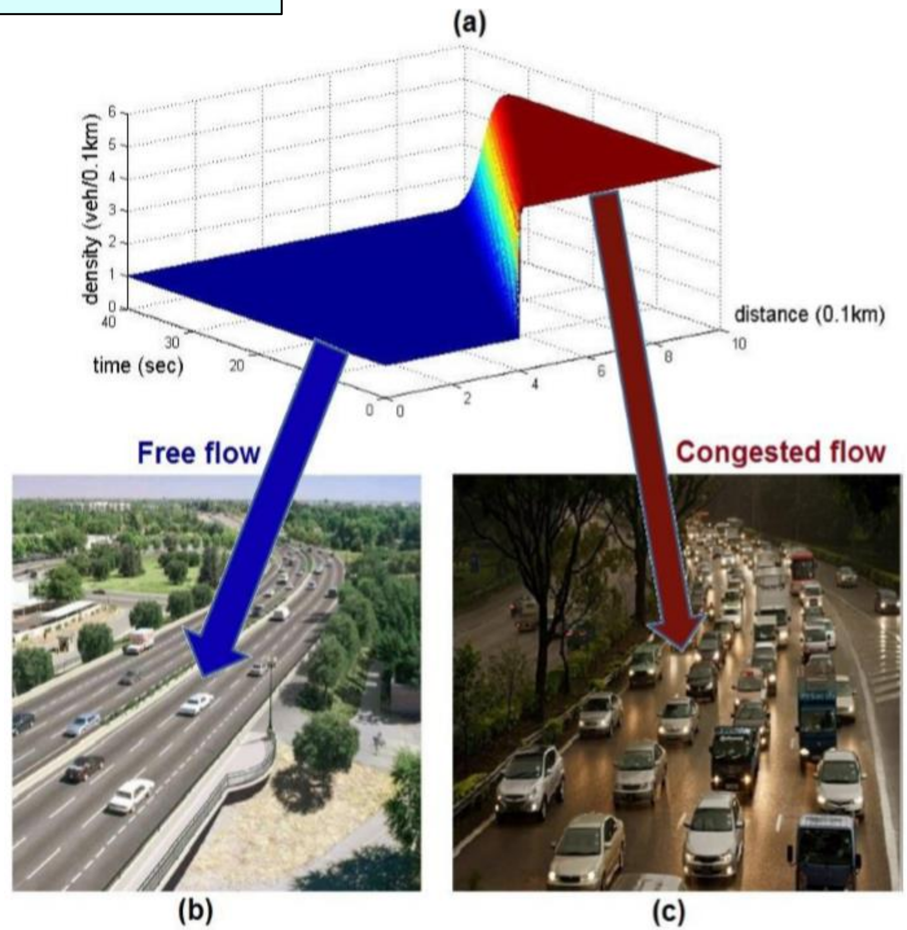

Fig.2. a) Free and congested flow states represented in dynamical system perspective. b) Corresponding real free flow state. c) Corresponding real congested flow. $c_0 = 6\frac{m}{s}$; $a = \frac{1}{14s}$; $v_f = 30\frac{m}{s}$ and $\rho_m = 0.2\frac{veh}{m}$

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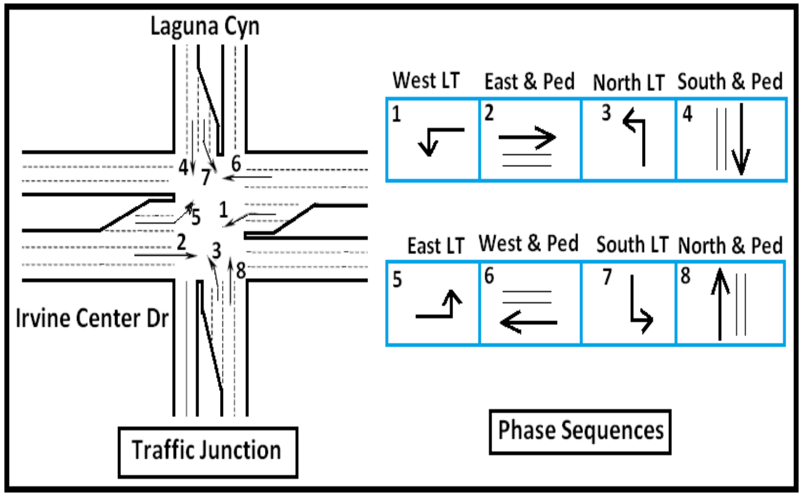
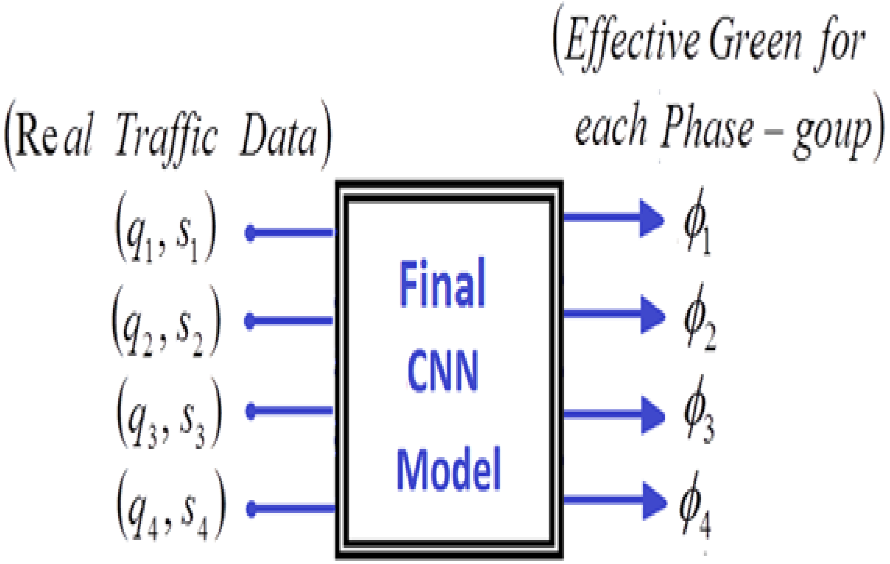
(Sample Result 3. Published [3])
Traffic Signals control & optimization

- Oscillators- based traffic control





- This CNN- platform is used to calculate the effective greens of all phase- groups in terms of the input traffic data.

$$\frac{d\Phi_i}{dt} = -\Phi_i + \sum_{j=1}^{M}\left[\hat{A}_{ij}\Phi_j + A_{ij}Y_j^* + B_{ij}u_j\right] + I_i$$

Results:
– Realtime capable
– Extremly flexible
– Robust system theory
PATENT APPLICATION !

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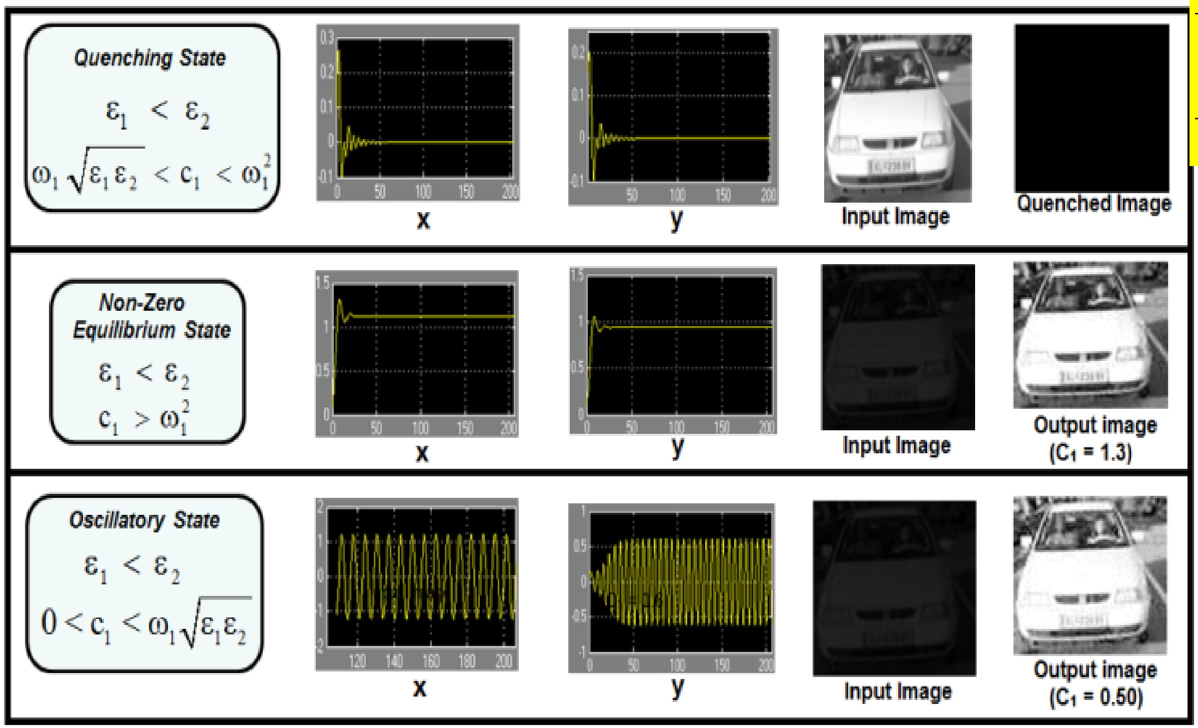
(Sample Result 4. Published [4])
Traffic sensors: Image Denoising & Conversion to binary images

- Robust and ultrafast low-level image processing in dynamically changing difficult visual environments

$$\frac{d^2\vec{x}}{dt^2} - \varepsilon_1(1-\vec{x}^2) + \omega_1^2\vec{x} = c_1\vec{y} + c_2\frac{d\vec{y}}{dt} \quad (1a)$$

$$\frac{d^2\vec{y}}{dt^2} + \varepsilon_2\frac{d\vec{y}}{dt} + \omega_2^2\vec{y} + c_0\vec{y}^3 = c_3\vec{x} + c_4\frac{d\vec{x}}{dt} \quad (1b)$$



Extremely useful for driver assistance systems

Performance

- ✓ Real-time processing
- ✓ Robustness
- ✓ Image sharpeness

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(Sample Result 5. Published [5]) Neuro-processor based TSP solver

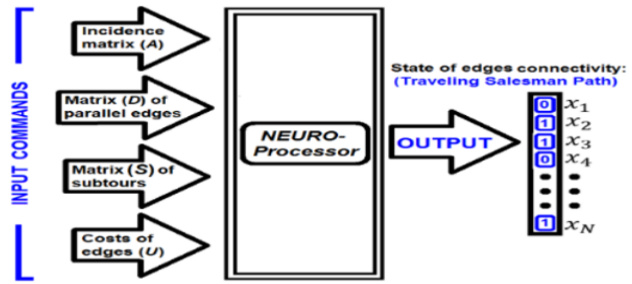

FIGURE 1. Synoptic representation of the neuro-processor based TSP solver concept. The four inputs are fundamental parameters of the graph under investigation. These inputs are used as external commands. The output x_i=1 expresses the belonging of an edge to the TSP solution, and x_i=0 otherwise.

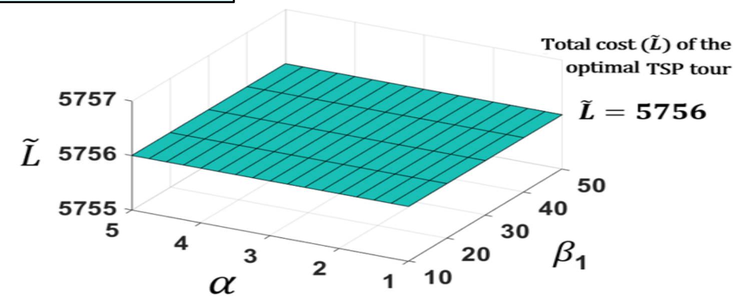

FIGURE 3. Results of the 2D bifurcation analysis in terms of α and β_1.in a graph of magnitude 24 and size 592 (see Figure 16). The total cost ($\tilde{L}$) of the TSP tour detected remains constant for all values of α and β_1 selected in windows $1 \leq \alpha \leq 5$ and $10 < \beta_1 \leq 50$. Further, it has been found through our various numerical simulations that $\tilde{L}$ corresponds to the global minimum. Thus, an optimality of the TSP tour is ensured in selected windows of α and β_1.

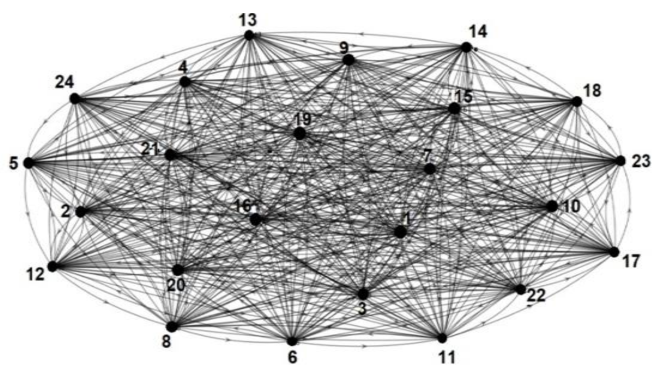

FIGURE 2. A complete graph network of magnitude "24" and size "552". Edges are bi-directional and their costs are obtained through the expression $c_i = i$ ($i = 1, 2, \ldots, 552$ stand for the indexes of edges). The optimal TSP tour is obtained as numerical solution of (15). The values of parameters used to solve (15) are defined as follows: $\beta_2 = 100$, $\beta_3 = 150$, $\beta_3^* = 50$, $\beta_4 = 10$, $\beta_5 = 0.045$, $h = 0.00025$ (step size) $1 \leq \alpha \leq 5$ and $10 < \beta_1 \leq 50$.

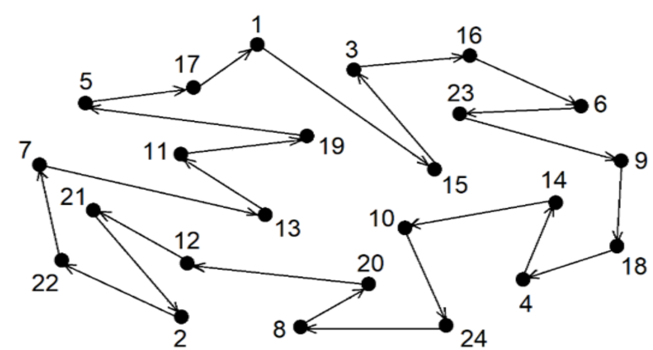

FIGURE 4 . TSP tour obtained according to the numerical solution $\vec{x}$ for the graph in Figure 16. The TSP tour detected corresponds to $x_i = 1$ (i=27, 32, 84, 85, 114, 115, 151, 160, 195, 200, 230, 243, 257, 276, 303, 312, 329, 340, 350, 369, 374, 385, 412 and 413). The cost of an edge is equal to the edge index. Thus using values of x_i in (9) leads to the total cost of TSP tour $\tilde{L} = \sum(i * x_i) = 5756$.

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References

[1]- J. C. Chedjou and K. Kyamakya, "A Road Traffic Behavioral Analysis: Mathematical Modelling and Numerical Simulation," Autonomous Systems, VDI Verlag GmbH, pp. 35 – 50, Düsseldorf, Germany, 2019.

[2]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, "Neural-Network-Based Calibration of Macroscopic Traffic Flow Models," in Recent Advances in Nonlinear Dynamics and Synchronization, Springer 2017.

[3]- J. C. Chedjou and K. Kyamakya, "Cellular Neural Networks Based Local Traffic Signals Control at a Junction/Intersection," IFAC- International Federation of Automatic Control, pp. 2034 - 2040, CESCIT 2012, Würzburg, Germany, 2012.

[4]- M. A. Latif, J. C. Chedjou, and K. Kyamakya, " Robust contrast enhancement by a coupled oscillatory paradigm: An application for visual sensors in transportation," COMPEL: The international journal for computation and mathematics in electrical and electronic engineering, 2011, 4, 1416-1432 .

[5]- J. C. Chedjou, K. Kyamakya, and N. A. Akwir, "An Efficient, Scalable and Robust Neuro-processor Based Concept for Solving Traveling Salesman Problems in Complex and Dynamically Reconfigurable Graph Networks," IEEE Access, vol. 8, pp. 42297 – 42324, March 2020.

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A didactic lecture on the topic:

"Mathematical modelling and Numerical simulation of road traffic on Highway"

(Macroscopic level of details)

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Literature

[1]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, “Neural-Network-Based Calibration of Macroscopic Traffic Flow Models,” in Recent Advances in Nonlinear Dynamics and Synchronization, Springer 2017.

[2]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, et al., “Traffic flow simulation: Comparison Between Finite Difference Mathod (FDM) and Method of Lines (MOL),” in XVIII International Symposium on Theoretical Electrical Engineering ISTET 15, Kolobrzeg, 2015.

[3]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, et al., “Traffic Characteristics on Highway: Bifurcation Analysis and Basins of Attraction,” in Proceedings INDS-2015.

[4]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, “Dynamic Optimization and CNN-Based Solving of ODEs and PDEs,” in XVIII International Symposium on Theoretical Electrical Engineering ISTET 15, Kolobrzeg, 2015.

[5]- N. A. Akwir, J. C. Chedjou and K. Kyamakya, “A Survey of Traffic Flow Modelling: Pros and Cons of traditional models,” Autonomous Systems. VDI Verlag, pp. 230–244, 2017.

[6]- N. A. Akwir, J. C. Chedjou, K. Kyamakya, et al., “A Fully Neurocomputing Based Traffic Modelling-and-Simulation Concept,” Autonomous Systems, VDI Verlag GmbH, pp. 131 – 150, 2018.

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"Didactic example: Application of a pedagogical approach on the subject/Topic entitled“

Mathematical modelling and numerical simulation of the complex behaviour of traffic flow on a road without ramps: The LWR’s Model

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Goals of traffic flow theory

- **The traffic flow theory** studies interactions between travelers (including pedestrians, cyclists, drivers, and their vehicles) and infrastructure (including highways, signage, and traffic control devices), with the aim of understanding and developing an optimal transport network to ensure the efficient movement of traffic and minimize traffic congestion and accidents.

Traffic stakeholders

- Travelers (including pedestrians, cyclists, drivers, and their vehicles), and infrastructure (including highways, signage, and traffic control devices) are the traffic stakeholders.

Macroscopic Traffic Flow Modeling

- Traffic states are characterized by the fundamental parameters flow, speed, and density.
- The traffic states estimation (TSE) is very important for traffic management. The TSE is based on the analysis of interactions between flow, speed, and density. The outcome of the analysis leads to the depiction of potential traffic states (e.g. under-saturated states, saturation state, oversaturated stated, shockwaves, refraction waves, rarefaction waves, synchronized states, chaotic states, etc.)

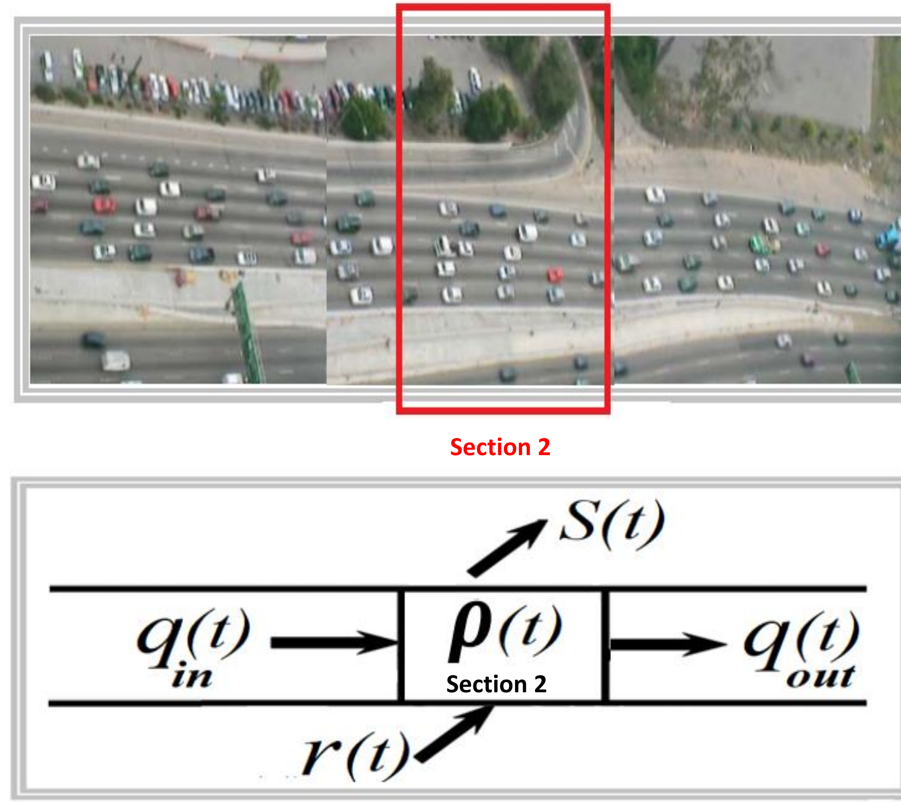


Fig. 1. Representation of traffic flow in a single section of the road segment. This representation considers a single section of the road segment as a dynamical system with inputs and outputs. The simple representation here is considered for the sake of facilitating the modeling procedure.

Traffic volume on roadway (1)

- **Definition 1:** The cumulative number of vehicles both in space and time is defined by the spatiotemporal function

$$N(x,t)$$

- **Flow:** Number of vehicles passing a given section of the road within a time period Δt

$$\frac{\Delta N(x,t)}{\Delta t} = q(x,t) \Rightarrow \frac{\partial N(x,t)}{\partial t} \approx q(x,t) \quad (1)$$

- **Density:** Number of vehicles that can be found in a road segment of length Δx. Justify the negative sign below, in the space dimension.

$$\frac{\Delta N(x,t)}{\Delta x} = -\rho(x,t) \Rightarrow \frac{\partial N(x,t)}{\partial x} \approx -\rho(x,t) \quad (2)$$

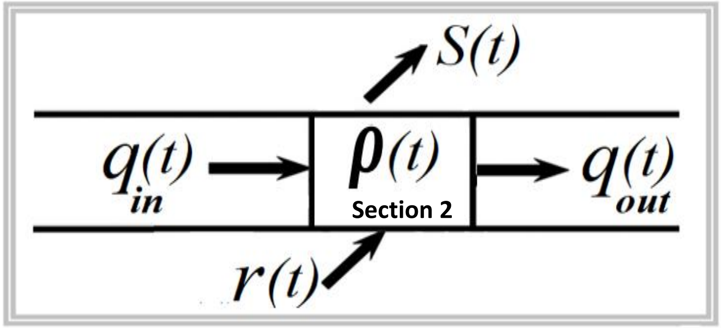


Fig. 1. Representation of traffic flow in a single section of the road segment. This representation considers a single section of the road segment as a dynamical system with inputs and outputs. The simple representation here is considered for the sake of facilitating the modeling procedure.

Traffic volume on roadway (2)

- **Comment 1a:** Let's note that in equation (1), the quantity $\Delta N(x,t)$ is positive ($\Delta N(x,t)>0$) since it is measured at a fixed-position (i.e., the spatial dimension is constant) and the temporal dimension is variable.
- **Comment 1b:** In contrast in equation (2), the quantity $\Delta N(x,t)$ is negative ($\Delta N(x,t)<0$) since it is measured at variable position (i.e., the spatial dimension is variable).
- **Comment 2:** Equations (1) and (2) show that the flow can be expressed as a function of the density.

$$\frac{\partial N(x,t)}{\partial t} \approx q(x,t) \;\; and \;\; \frac{\partial N(x,t)}{\partial x} \approx -\rho(x,t) \quad (3)$$

⇩

$$\frac{\partial N(x,t)}{\partial t} = \mathcal{F}\left(-\frac{\partial N(x,t)}{\partial x}\right) \quad (4)$$

Section 2

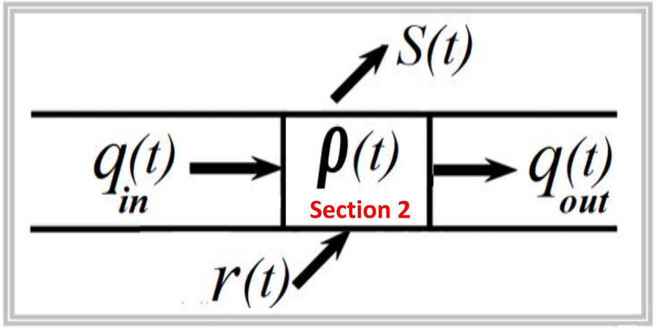


Fig. 1. Representation of traffic flow in a single section of the road segment. This representation considers a single section of the road segment as a dynamical system with inputs and outputs. The simple representation here is considered for the sake of facilitating the modeling procedure.

Traffic volume on roadway (3)

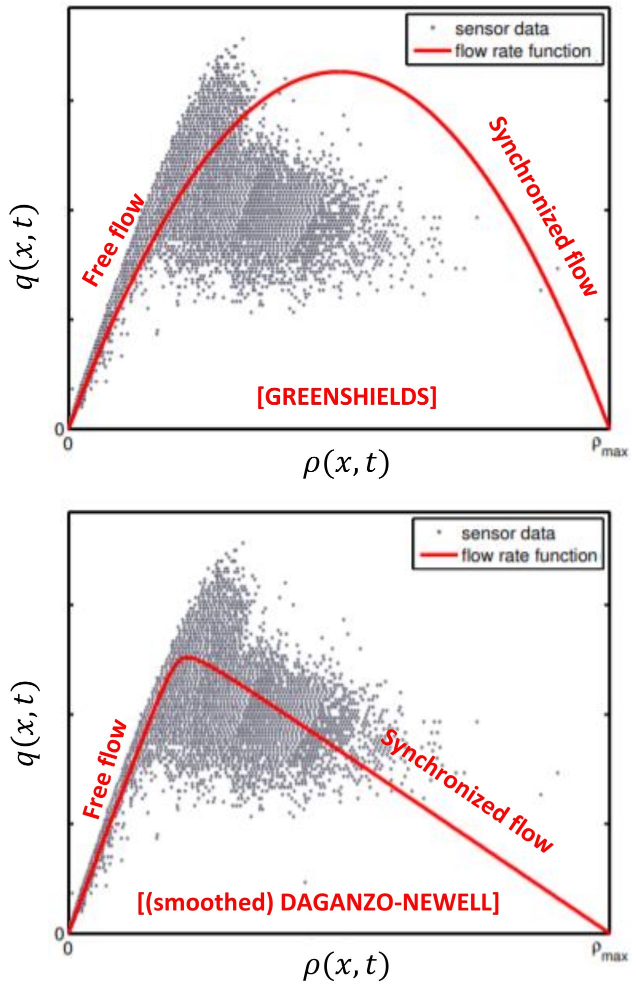
- **Comment 3:** Considering the fundamental diagram in Figs. 2, equation (4) can be approximated by the expression (5).

$$\frac{\partial N(x,t)}{\partial t} = \mathcal{F}\left(-\frac{\partial N(x,t)}{\partial x}\right) \quad (4)$$

⇩

$$\frac{\partial N(x,t)}{\partial t} + \alpha \frac{\partial N(x,t)}{\partial x} = 0 \quad (5)$$

- **Comment 4:** The coefficient α corresponds to the speed. Therefore, <u>**three possibilities exist for α**</u>.
- **Comment 5:** According to Figs. 2, <u>**the speed α can be constant or variable**</u>. These conditions on the speed <u>α</u> depend on specific regions in Figs. 2.



Figs. 2. Fundamental diagrams: "flow versus density". The diagrams are used to approximate equation (4) by equation (5). The traffic states under which the approximation is valid must be clearly described here.

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Traffic volume on roadway (4)

- **Comment 6:** Assuming the traffic at equilibrium (i.e. α is a constant parameter), equation (5) can be solved analytically.

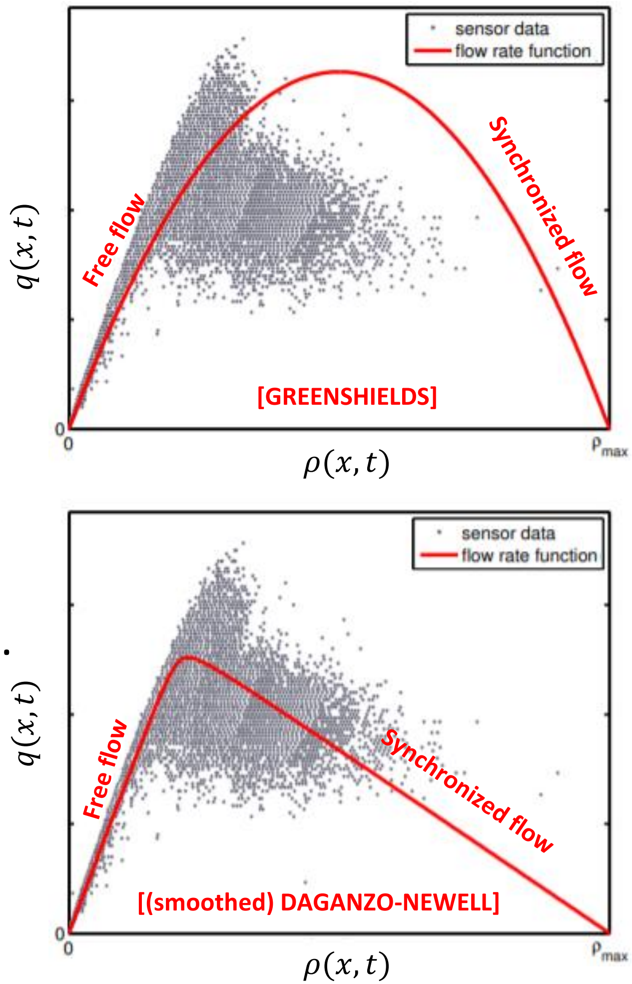
$$\frac{\partial N(x,t)}{\partial t} + \alpha \frac{\partial N(x,t)}{\partial x} = 0 \quad (5)$$

- **Comment 7:** The analytical solution of equation (5) can be obtained using the method of separation of variables. This method consists of expressing the cumulative number of vehicles $N(x,t)$ in the form (6).

$$N(x,t) = f(x).\, g(t) \quad (6)$$

- **Comment 8:** Using (5) and (6) leads to the following analytical solution:

$$\begin{cases} f(x) = c_1 e^{\lambda x} \\ g(t) = c_2 e^{-\alpha \lambda t} \end{cases} \quad (7)$$



Figs. 2. Fundamental diagrams: "flow versus density". The diagrams are used to approximate equation (4) by equation (5). The traffic states under which the approximation is valid must be clearly describe here.

Traffic volume on roadway (5)

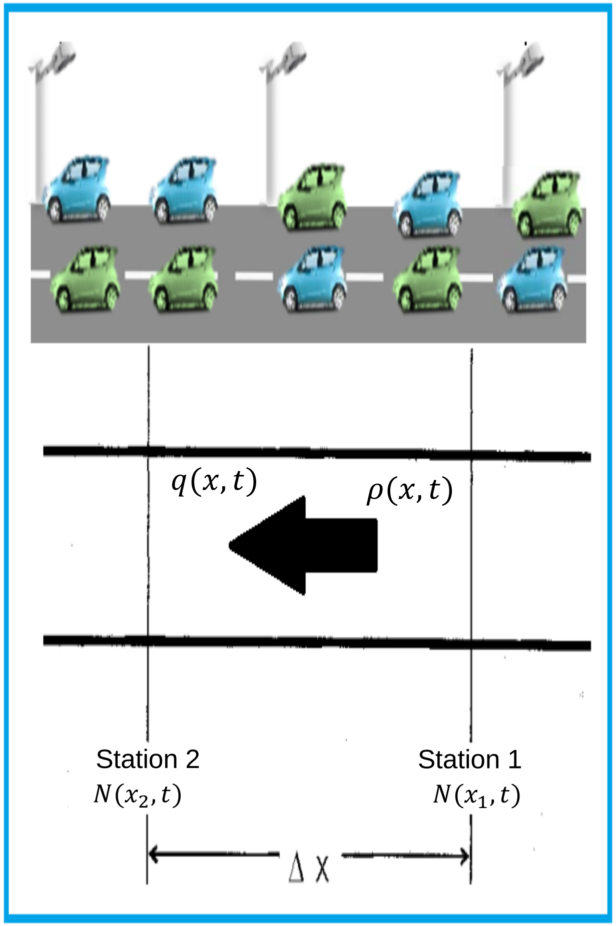
- **Comment 9:** The constants c_1 and c_2 are obtained through the boundary and initial conditions.

- **Comment 10:** Finally the spatiotemporal evolution of the cumulative number of vehicles $N(x,t)$ is expressed into the analytical form (8).

$$N(x,t) = c_1 c_2 e^{\lambda(x-\alpha t)} \quad (8)$$

The conservation law in traffic (1)

- According to (8), the cumulative number of vehicles at stations 1 and 2 are denoted by $N(x_1,t)$ and $N(x_2,t)$. Note that the traffic counting is performed at stations 1 & 2. The distance between the two stations is Δx. We assume that there is no ramps between the two stations (i.e., no incoming car and no outgoing car).



Figs. 3. Traffic flow on the road (between two stations)

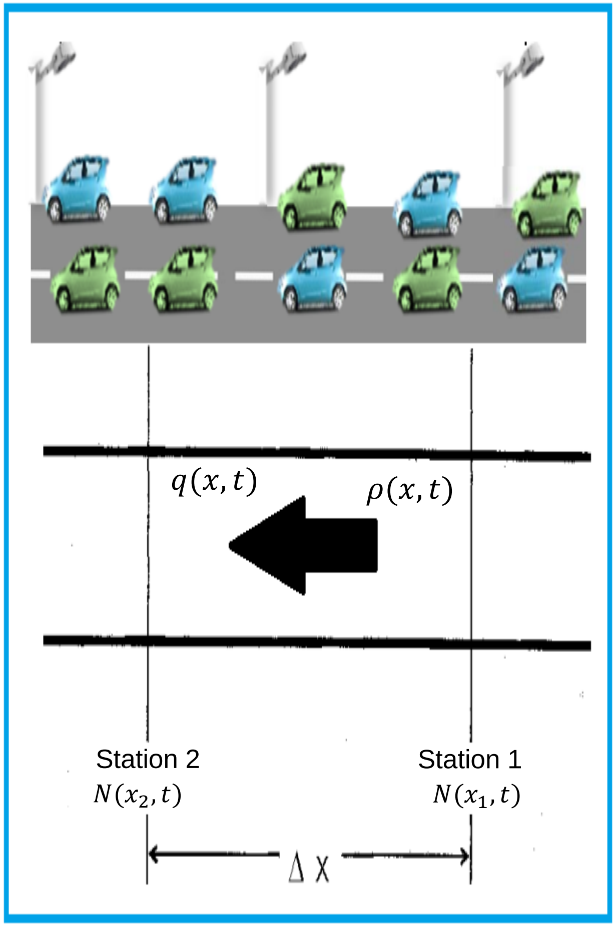
The conservation law in traffic (2)

- The traffic flows at stations 1 & 2 are expressed by (9). In (9), Δt is the duration of traffic counting at each station.

$$\begin{cases} q(x_1,t) = \dfrac{N(x_1,t)}{\Delta t} \\ q(x_2,t) = \dfrac{N(x_2,t)}{\Delta t} \end{cases} \qquad (9)$$

- **Comment 1.** Let's note that considering the conservation law, we have $N(x_1,t) > N(x_2,t)$ since all vehicles $N(x_1,t)$ passing through station 1 are likely to reach station 2, however with a delay corresponding to the travel-time experienced by each vehicle when moving from station 1 to station 2. Therefore the variation in traffic counting is expressed by (10).

$$\Delta N = N(x_2,t) - N(x_1,t) < 0 \qquad (10)$$



Figs. 3. Traffic flow on the road (between two stations)

The conservation law in traffic (3)

- Using (9) the variation of flow Δq can be calculated as follows:

$$\Delta q(x,t) = \frac{\Delta N}{\Delta t} = \frac{[N(x_2,t) - N(x_1,t)]}{\Delta t} < 0$$

$$\Downarrow$$

$$\Delta N = \Delta q . \Delta t < 0 \qquad (11)$$

- The traffic density between stations 1 & 2 is **a positive quantity** that can be expressed as follows (see (12)):

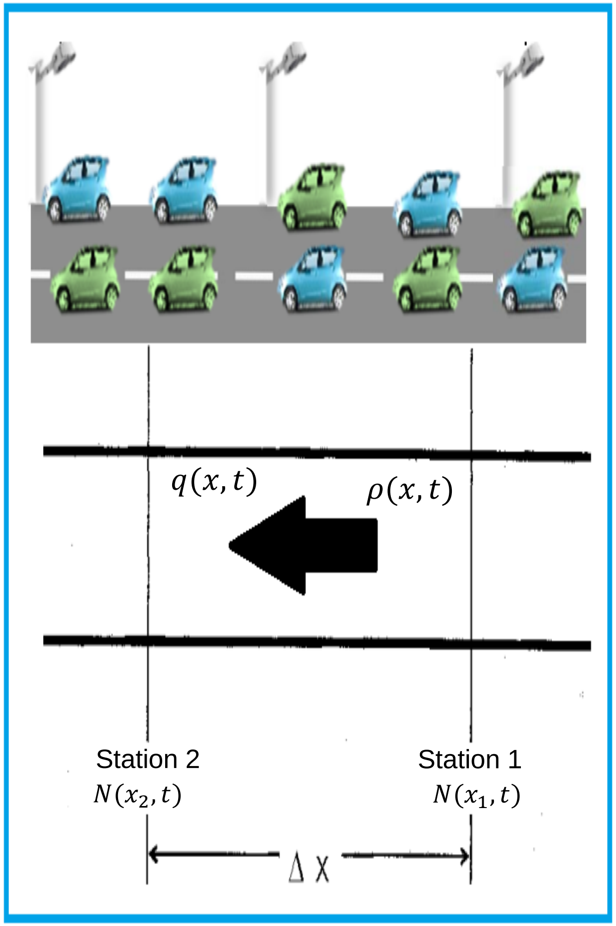
$$\Delta \rho(x,t) = \frac{-\Delta N}{\Delta x} = \frac{-[N(x_2,t) - N(x_1,t)]}{\Delta x} > 0$$

$$\Downarrow$$

$$\Delta N = -\Delta \rho . \Delta x < 0 \qquad (12)$$

- Using (11) and (12), the following expression is obtained:

$$\frac{\Delta \rho(x,t)}{\Delta t} + \frac{\Delta q(x,t)}{\Delta x} = 0 \qquad (13)$$



Figs. 3. Traffic flow on a road section of length ΔX

The conservation law in traffic (4)

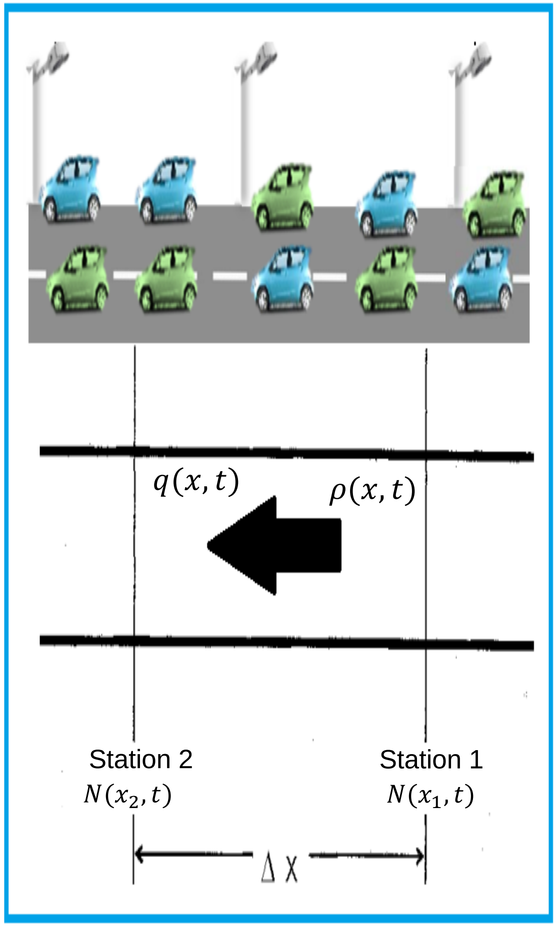
- Assuming infinitesimal (i.e., very small) variations in Δt and Δx consequently results to small variations in $\Delta\rho(x,t)$ and $\Delta q(x,t)$. Therefore equation (13) can be approximated by the continuous form (14) without any loss of generality.

$$\frac{\Delta\rho(x,t)}{\Delta \mathrm{t}} + \frac{\Delta q(x,t)}{\Delta x} = 0 \qquad (13)$$

$$\Downarrow$$

$$\frac{\partial\rho(x,t)}{\partial \mathrm{t}} + \frac{\partial q(x,t)}{\partial x} = 0 \qquad (14)$$

- **Comment 2.** Equation (14) is used to model the spatio-temporal continuous evolution of the flow, speed, and density on a road segment of length Δx in the absence of ramps (no exit ramp, and no In-ramp).
- **Comment 3.** Equation (14) models the law of conservation of a traffic stream. This fundamental equation is called (see literature): **"the conservation or continuity equation".**



Figs. 3. Traffic flow on a road section of length ΔX

The conservation (or continuity) equation [LWR's Model 1956] (1)

- The continuity (or conservation) equation was **developed in 1956** by Lighthill, Whitham and Richard's (see Eq. (14)). The acronym of **"LWR's model"** was further assigned to (14).

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \Leftrightarrow \frac{\partial \rho(x,t)}{\partial t} + \frac{\partial[\rho u(\rho)]}{\partial x} = 0 \quad (14)$$

- **Comment 1.** Eq. (14) cannot be solved as it contains two variables (u, ρ). Therefore the commonly used idea consists of modeling the speed(u) as a function of the density (ρ). The mathematical formula obtained (see the empirical formulas below) is based on the use of sensor data (i.e., experimental measurements). The procedure consists of **plotting sensor data** and using both **stationary equilibrium** and **strong equilibrium** to obtain the empirical analytical model of $f(\rho)$ and $u(\rho)$. The procedure is explained in the next slide.

$$f(\rho) = \frac{\rho}{\rho_{max}}\left(1 - \frac{\rho}{\rho_{max}}\right)u_{max}$$ (Empirical function $f(\rho)$)

- At very low occupancy ➡ $u(0) = u_{max}$
- At complete jam ➡ $u(\rho_{max}) = 0$
- In between ➡ $u(\rho) = u_{max}(1 - \rho/\rho_{max})$

The conservation (or continuity) equation [LWR's Model 1956] (2)

1. History

The first beginnings for traffic flow descriptions on a highway are derived fi observations by Greenshields, firstly shown to the public exactly 75 years ago (P of the 13th Annual Meeting of the Highway Research Board, Dec. 1933). He car out tests to measure traffic flow, traffic density and speed using photogra measurement methods for the first time. A short look on his CV shows Greenshields started his career as traffic engineering scientist with this publica which leads to a PhD-thesis at the University of Michigan in 1934.

Bruce D. Greenshields

- born in Winfield, Kansas; grew up in Blackwell, Oklahoma
- graduate of University of Oklahoma; earned Masters Degree in Civil Engineering Univ. of Michigan
- 1934 Doctorate in Civil Engineering from University of Michigan
- taught at different Universities
- wrote numerous articles on traffic behavior and highway safety
- pioneer in the use of photography relating to traffic matters and in applying mathematics to traffic flow
- invented the „Drivometer“
- 1956 joined the University of Michigan faculty and was Acting Director of the Transportation Institute there
- 1966 retirement
- then returned to Washington and was a traffic consultant to various federal agencies
- Dr. Greenshields received the Matson Memorial Award in 1976

Fig. 1.: CV of Bruce Douglas Greenshields

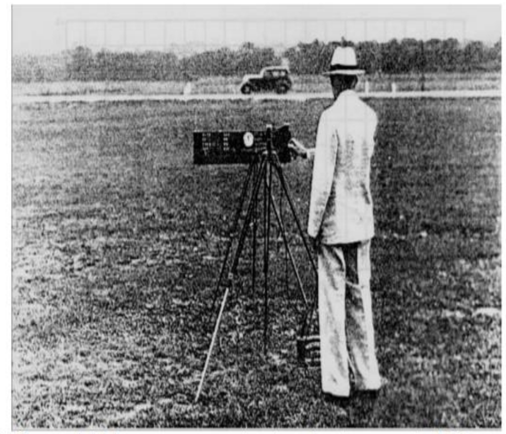


Fig. 2: Greenshields measurement set up for the reported 75 years ago

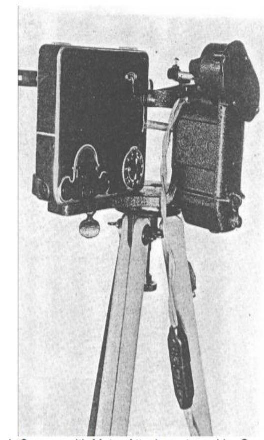


Fig. 4: Camera with Motor Attachment used by Greenshields

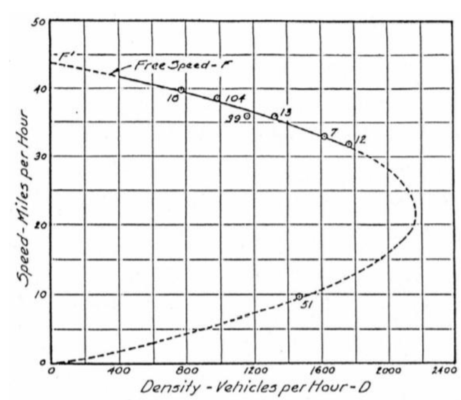


Fig. 6: The first Fundamental Diagram as v-q Diagram

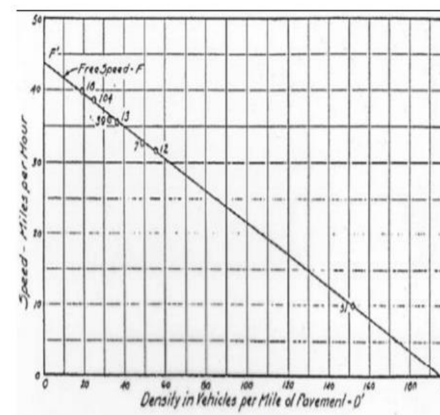
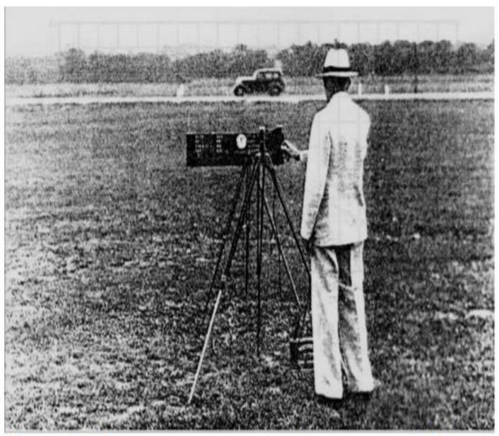


Fig. 5: Speed Density Relation V (Greenshield 1934)

The conservation (or continuity) equation [LWR's Model 1956] (3)

**Foundations of Traffic Flow Theory I:
Greenshields' Legacy – Highway Traffic**

Prof. Dr. Reinhart D. Kühne
German Aerospace Center, Transportation Studies, Berlin, Germany



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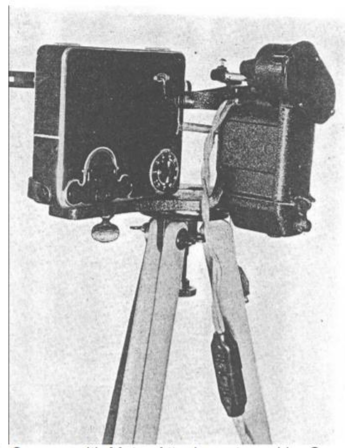
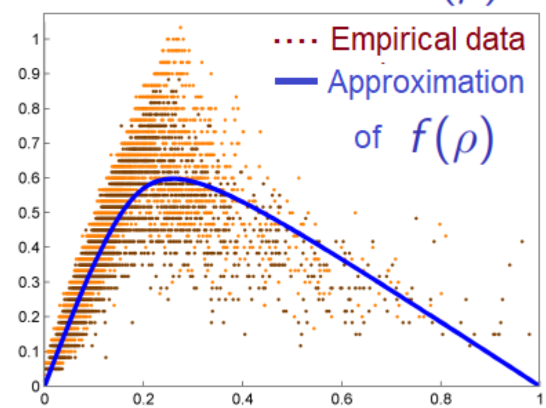


Fig. 4: Camera with Motor Attachment used by Greenshields

A more realistic $f(\rho)$



Stationary: $\begin{cases} \frac{\partial \rho}{\partial t} = 0 \\ \frac{\partial u}{\partial t} = 0 \end{cases}$

Strong equilibrium: $\begin{cases} \frac{\partial \rho}{\partial x} = 0 \\ \frac{\partial u}{\partial x} = 0 \end{cases}$

$$f(\rho) = \frac{\rho}{\rho_{max}}\left(1 - \frac{\rho}{\rho_{max}}\right)u_{max}$$

(Empirical function $f(\rho)$)

- At very low occupancy ➡ $u(0) = u_{max}$
- At complete jam ➡ $u(\rho_{max}) = 0$
- In between ➡ $u(\rho) = u_{max}(1 - \rho/\rho_{max})$

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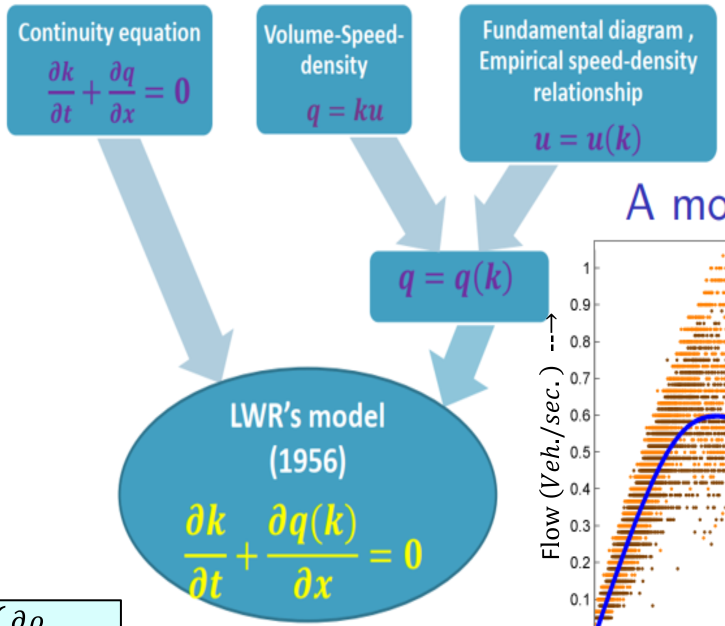
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

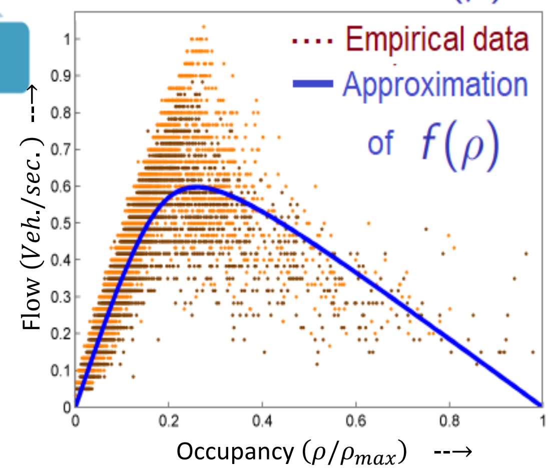
The conservation (or continuity) equation [LWR's] (4)

Remark: In all equations below, the variable k stands for the density $\rho(x,t)$ in equation (13)

- **How to obtain the empirical analytical/mathematical model?**
  - ✓ Step 1. We plot the sensor data (experimental data).
  - ✓ We use the **stationary equilibrium** and the **strong equilibrium concepts** to obtain the trajectory of the curve in blue color. This curve is the approximation of $f(\rho)$).



A more realistic $f(\rho)$



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$$f(\rho) = \frac{\rho}{\rho_{max}}\left(1 - \frac{\rho}{\rho_{max}}\right)u_{max}$$ (Empirical function $f(\rho)$)

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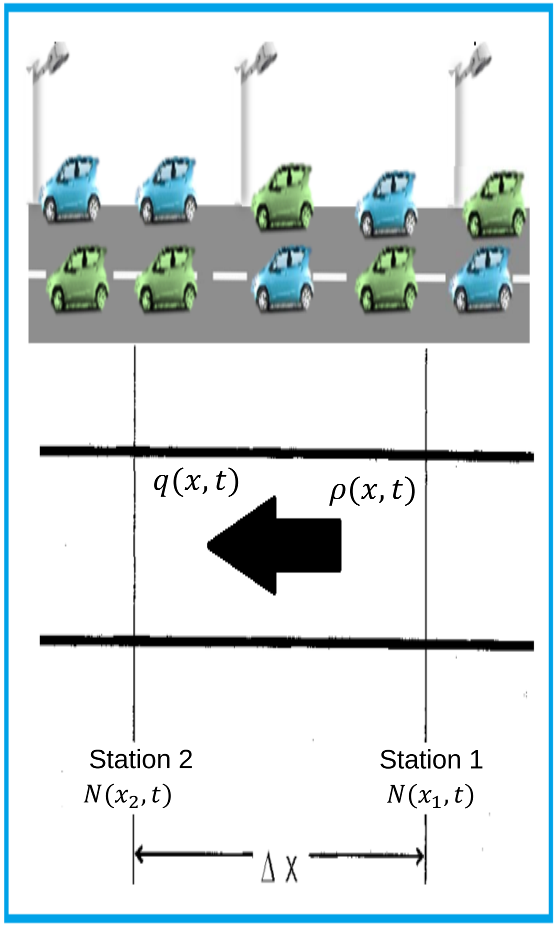
Pros and Cons of the continuity equation

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial[\rho u(\rho)]}{\partial x} = 0 \quad (14)$$

- **Pros.** Equation (14) can be used to model the following interesting dynamics of traffic flow: **Shockwaves** (both formation and propagation)**, Refraction waves, Rarefaction waves,** just to name a few.
- The LWR model quite nicely explains the shape of traffic jams (vehicles run into a shock).

- **Cons.** Equation (14) assumes the **same type of vehicles**, **Constant speed**, **Infinite acceleration**. Further, Eq. (14) is unable to model the following: **Stop-and-go-waves**, **Traffic hysteresis**, **Breakdown**, **Bottleneck**, just to name a few.
- Shortcomings of LWR: Perturbations never grow (\maximum principle"). Thus, phantom traffic jams cannot be explained with LWR. And neither can multi-valued FD.



Figs. 3. Traffic flow on a road section of length ΔX

Numerical simulation of the LWR's model (1)

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial [\rho u(\rho)]}{\partial x} = 0 \qquad (14)$$

- Choice of MATLAB for scientific computing
- Choice of the method (MOL, FDM, FEM, FVM)
- Discretization method (Euler, Taylor, Upwind, FTCS, Lax, Lax- Friedrichs, Lax- Wendroff, etc.)
- Parameters settings
- Initial condition
- Boundary conditions
- Numerical scheme used
- Simulation results
- Comment of simulation results
- Concluding remarks
- Introduction of the next investigation: Advantage

Schemes	Computing speed	Accuracy
Method Of Line (MOL)	Fast	Better accuracy
Finite Difference Method (FDM)	Fast	Less accuracy
Finite Element Method (FEM)	Fast	High accuracy
Finite Volume Method (FVM)	Slow	Very high accuracy

Numerical simulation of the LWR's model (2)

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial [\rho u(\rho)]}{\partial x} = 0 \quad (14)$$

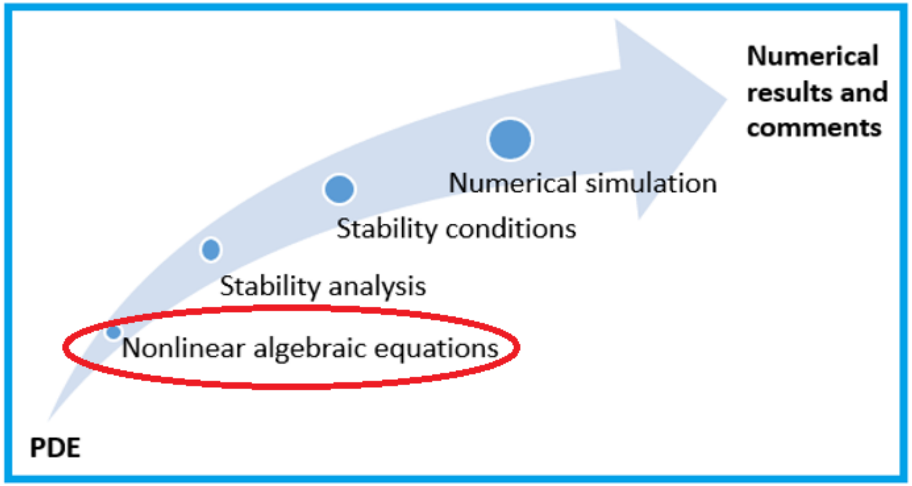


Fig. 1: Full/Complete steps of the FDM

❑ Choice of the method: FDM

- ✓ The Finite Difference Method (FDM) transforms the Partial Differential Equation (PDE) into a set of coupled discrete algebraic equations.
- ✓ The numerical solution of the coupled discrete algebraic equations is obtained using the control flow statements such as "For- loops", "While-loops", etc.

Table 1. Formulas to approximate the derivatives

	With respect to x	With respect to t
Forward difference	$\frac{\partial f_i^n}{\partial x} \approx \frac{f_{i+1}^n - f_i^n}{\Delta x}$	$\frac{\partial f_i^n}{\partial t} \approx \frac{f_i^{n+1} - f_i^n}{\Delta t}$
Central difference	$\frac{\partial f_i^n}{\partial x} \approx \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x}$	$\frac{\partial f_i^n}{\partial t} \approx \frac{f_i^{n+1} - f_i^{n-1}}{2\Delta t}$
Backward difference	$\frac{\partial f_i^n}{\partial x} \approx \frac{f_i^n - f_{i-1}^n}{\Delta x}$	$\frac{\partial f_i^n}{\partial t} \approx \frac{f_i^n - f_i^{n-1}}{\Delta t}$

Numerical simulation of the LWR's model (3)

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial[\rho u(\rho)]}{\partial x}=0 \quad (14)$$

- **Choice of the method: MOL**
  - ✓ The Method of Lines (MOL) transforms the Partial Differential Equation (PDE) into a set of coupled Ordinary Differential Equations (ODEs).
  - ✓ The numerical solution of the coupled ODEs is obtained by coding the coupled ODEs into a MATLAB-script. This script is further analyzed by the ODE-MATLAB-SOLVERS (e.g., ode45, ode23, ode15s, ode113, ode23t, ode23s, ode23tb, etc.)

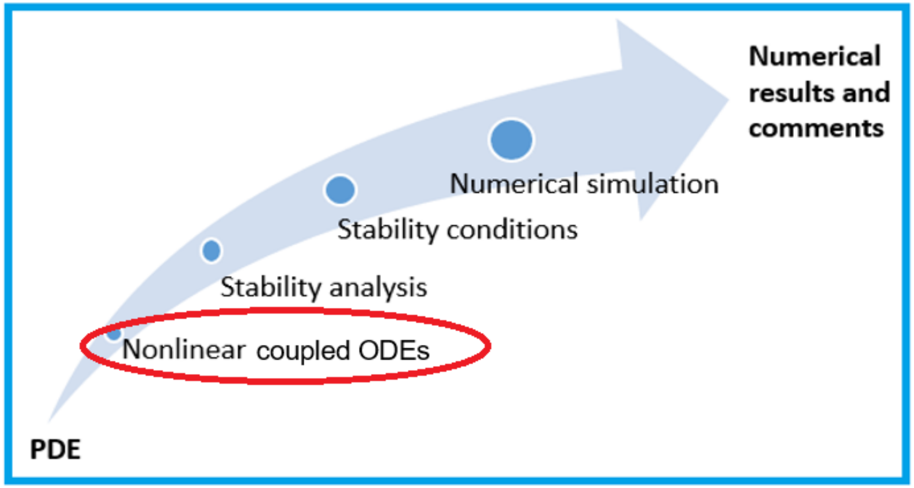


Fig. 1: Full/Complete steps of the MOL

Table 1. Formulas to approximate the derivatives

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Central difference	$\frac{\partial f_i^n}{\partial x}\approx\frac{f_{i+1}^n-f_{i-1}^n}{2\Delta x}$	$\frac{\partial f_i^n}{\partial t}\approx\frac{f_i^{n+1}-f_i^{n-1}}{2\Delta t}$
Backward difference	$\frac{\partial f_i^n}{\partial x}\approx\frac{f_i^n-f_{i-1}^n}{\Delta x}$	$\frac{\partial f_i^n}{\partial t}\approx\frac{f_i^n-f_i^{n-1}}{\Delta t}$

Numerical simulation of the LWR's model (4)

Choice of the discretization scheme: LAX

- ✓ Various discretization schemes are proposed (see literature and also table below) for solving PDEs.
- ✓ The choice of a discretization scheme significantly affects the accuracy, the robustness, and the stability/convergence of the numerical framework used for solving the Partial Differential Equation (PDE).

$$\frac{\partial \rho(x,t)}{\partial \mathrm{t}} + \frac{\partial[\rho u(\rho)]}{\partial x} = 0 \quad (14)$$

Table 4. Discretization schemes with respect to time and space (spatiotemporal discretization)

Schemes	Time	Space
FTCS Method	Forward in Time (FT)	Central in Space (CS)
Upwind Methods	Forward (positive velocity) Forward (negative velocity)	Backward (positive velocity) Forward (negative velocity)
The Lax Method	Take case of FTSC and replace u_i^j by the average in space, i.e., $u_i^j = \frac{u_{i+1}^j + u_{i-1}^j}{2}$	
The Lax-Wendroff Method	Forward in Time (FT)	$\frac{\partial u}{\partial x} = \frac{u_{i+1/2}^{j+1/2} - u_{i-1/2}^{j+1/2}}{\Delta x}$

Numerical simulation of the LWR's model (5)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step1.** We solve the LWR's under the empirical speed-density of the Greenshields Model

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial[\rho u(\rho)]}{\partial x} = 0 \quad (14) \qquad \text{and} \qquad u = \left[u_f - \frac{u_f}{k_j}\rho\right] \quad (15)$$

  - ✓ **Step2.** We transform (14) and (15) into a PDE with a single variable ρ (see (16)). Please note that this step is not mandatory as we can also write an algorithm to solve (14) and (15) directly without the need of a preliminary transformation.

$$\frac{\partial \rho(x,t)}{\partial t} + \left[u_f - 2\frac{u_f}{k_j}\rho(x,t)\right]\frac{\partial \rho(x,t)}{\partial x} = 0 \quad (16)$$

  - ✓ The next step consists of choosing the discretization scheme for solving (16).

Numerical simulation of the LWR's model (6)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step3.** We use the LAX- METHOD for solving (16).

$$\frac{\partial \rho(x,t)}{\partial t} + \left[u_f - 2\frac{u_f}{k_j}\rho(x,t)\right]\frac{\partial \rho(x,t)}{\partial x} = 0 \quad (16)$$

Applying the LAX- METHOD to (16), the discrete expression (17) is obtained. In (17), the index "n" corresponds to the discrete time while the index"i“ corresponds to the discrete space.

(Forward in time) (Central in space) (Average in space dimension)

$$\frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} + \left[u_f - 2\frac{u_f}{k_j}\rho_i^n\right]\frac{\rho_{i+1}^n - \rho_{i-1}^n}{2\Delta x} = 0 \quad \text{and} \quad \rho_i^n = \frac{\rho_{i+1}^n + \rho_{i-1}^n}{2} \quad (17)$$

✓ Combining the two expressions in (17), the spatiotemporal evolution of the density ρ_i^{n+1} can be expressed into the following discrete form:

$$\rho_i^{n+1} = \frac{\rho_{i+1}^n + \rho_{i-1}^n}{2} - \left(\frac{\Delta t}{2\Delta x}\right)\left[u_f - 2\left(\frac{u_f}{k_j}\right)\frac{\rho_{i+1}^n + \rho_{i-1}^n}{2}\right](\rho_{i+1}^n - \rho_{i-1}^n) \quad (18)$$

Numerical simulation of the LWR's model (7)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step4.** MATLAB Script for the numerical solution of the discrete equation (18). → P1

```
%% SOLUTION OF THE "ADVECTION PDE USING THE UPWIND DISCRETIZATION METHOD/SCHEME"
%% ******************** Ut+cUx=0 ********************
clear all; clc;


%% Parameters needed to solve the Advection equation using the Upwindmethod
%%% TIME DIMENSION %%%
Tmax =50; % Maximum time
maxt1 =300; % Number of time steps
dt= Tmax/maxt1; % Step for Time Discretization
%%% SPACE DIMENSION %%%
Lmax = 15; % Maximum length
maxt2=204; % Number of space steps
dx = (Lmax/maxt2); % Step for Space Discretization
%%% COEFFISCIENTS/PARAMETERS OF THE EQUATION UNDER INVESTIGATION%%%
%c=0.5;
%%% STABILITY CONDITION FOR THE "UPWING DISCRETIZATION METHOD/SCHEME" %%%
%%% ******************** c*(dt/dx)<=1 ********************;
```

Numerical simulation of the LWR's model (8)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step4.** MATLAB Script for the numerical solution of the discrete equation (18). → P2

```
%% Initial value of the function u (amplitude of the wave)
for i = 1:(maxt2+1)
x(i) = (i-1)*dx;
%u(i,1)=exp( -10*(x(i)-2)^2 ) ;
u(i,1)=5;
%u(i,1)=12;
%u(i,1)=20;

end
```

```
%% Value of the amplitude at the boundary at any time
for n=1:(maxt1+1)
u(1,n) = 0;
u(maxt2+1,n) = u(i,1);
time(n) = (n-1)*dt;
end
```

Numerical simulation of the LWR's model (9)

- All steps of the Numerical solving of the LWR's Model (14)
  - ✓ **Step4.** MATLAB Script for the numerical solution of the discrete equation (18). → P3

```
%% Implementation of the Lax method
%% VERY IMPORTANT: THE INDEX "n" MUST BE ITERATED STARTING AT "1"
for n=1:maxt1 % Time loop
for i=2:maxt2 % Space loop

%uf=0.1667;
%uf=0.2222;
uf=0.2778;

kj=25;

%%% For the LAX- METHOD/SCHEME, the term u(i,n)is being replaced by
%% an average over its two neighbours.
u(i,n+1) = (1/2)*( u(i+1,n)+ u(i-1,n) ) +...
- (dt/(2*dx))*( uf - 2*(uf/kj)*( (1/2)*( u(i+1,n)+ u(i-1,n) ) ) )*(u(i+1,n) - u(i-1,n));
end
end
```

Numerical simulation of the LWR's model (10)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step4.** MATLAB Script for the numerical solution of the discrete equation (18). → P4

```
%% Graphical representations of the spatiotemporal evolution of the density on the
%% road.
figure(1)
mesh(x,time,u')

%% Graphical representations of the spatial evolution of the density (at specific
%% time slots).
figure(2)
plot(x,u(:,50),'-',x,u(:,100),'-',x,u(:,150),'-', x,    +...
u(:,200),'-',x,u(:,250),'-',x,u(:,300),'-', 'Linewidth', 5)


%% Graphical representations of the temporal evolution of the density (at specific
%% position on the road).
figure(3)
plot(time,u(204/6,:),'-',time,u(2*204/6, :),'-',time,u(3*204/6, :),'-', time, +...
u(4*204/6, :),'-', time,u(5*204/6, :),'-', time,u(6*204/6, :),'-', 'Linewidth', 5)

%%% ******************** END OF THE MATLAB SCRIPT ********************
```

Numerical simulation of the LWR's model (11)

- **CASE 1:** The parameter- settings used for the numerical simulation are defined as follows:

$$\boldsymbol{\rho(x, t_0) = 5Veh/(0.1km)}$$

$$\rho(x_0, t) = 0$$

$$\rho(x_{max}, t) = \rho(x, t_0)$$

$$u_f = 100Km/h$$

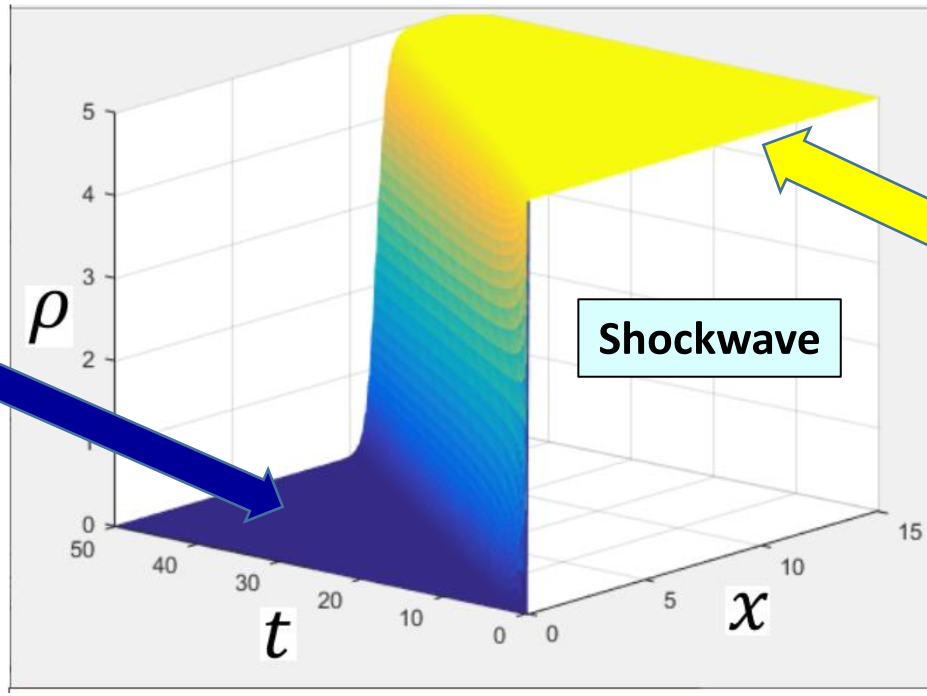
$$k_j = 25Veh/(0.1km)$$

Unit of length=0.1Km=100m

Numerical simulation of the LWR's model (12)

- ❑ **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 1:** Spatiotemporal evolution of the <u>**density**</u>. → Fig. 1

Low occupancy (Under-saturation)



Shockwave

High occupancy (Over-saturation)

- ❑ **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- ❑ **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

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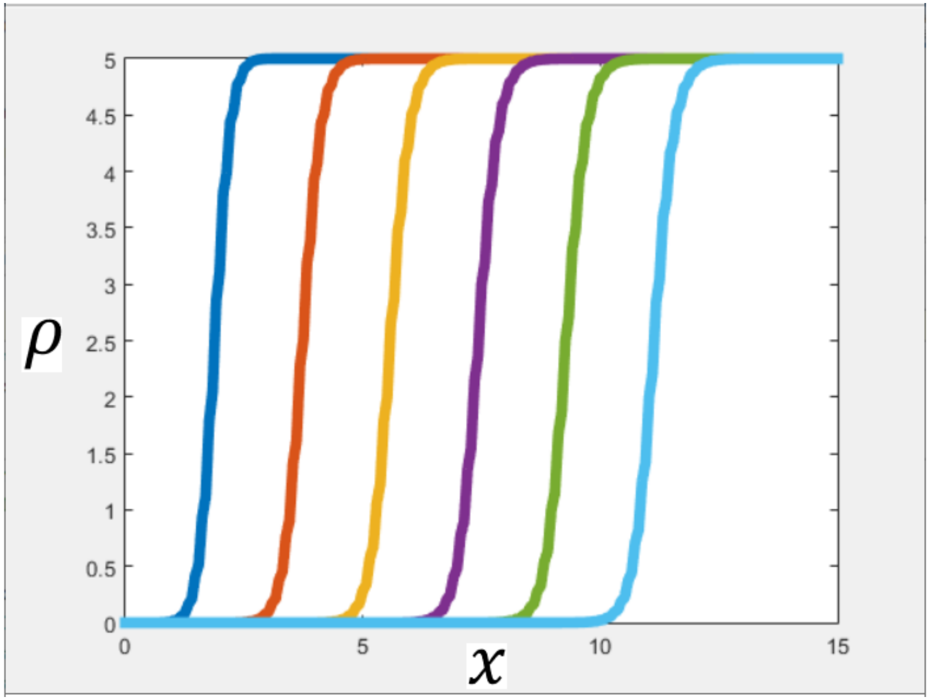
Numerical simulation of the LWR's model (13)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 2:** Spatial evolution of the density (at fixed time- slots) . → Fig. 2

Propagation of shockwave

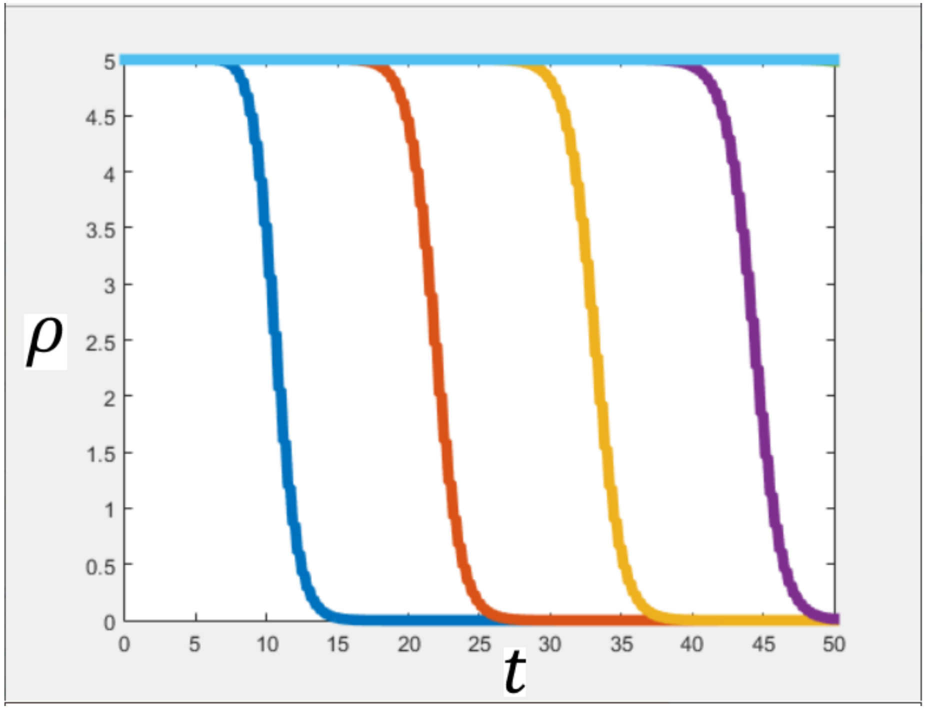


Propagation of shockwave

- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (14)

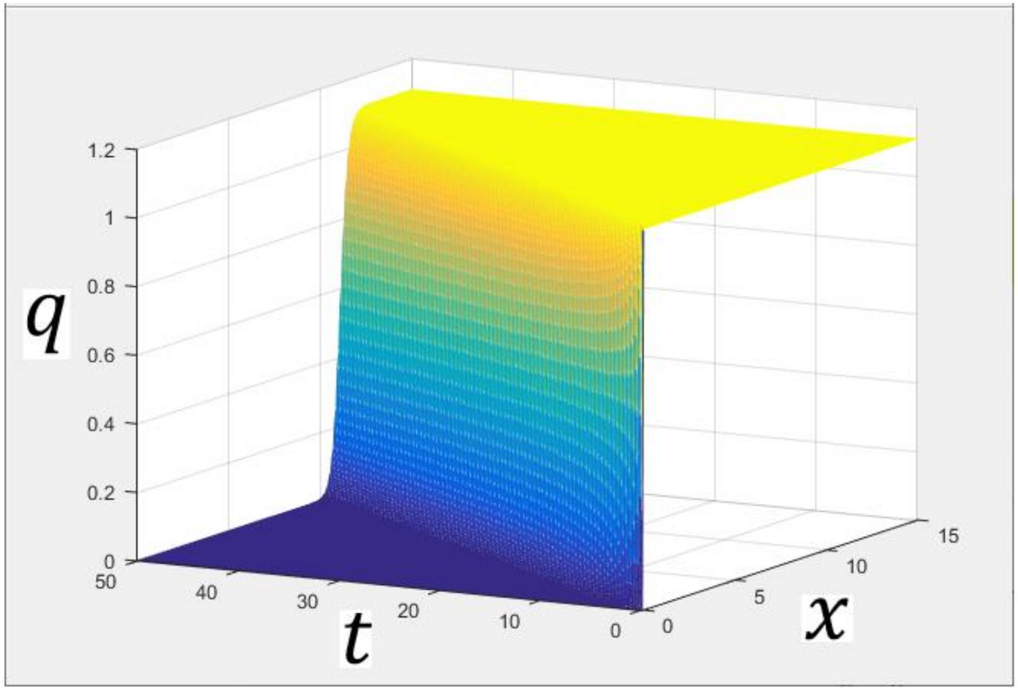
- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 3:** Temporal evolution of the density (at fixed positions). → Fig. 3



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (15)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 4:** Spatiotemporal evolution of the flow . → Fig. 4



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

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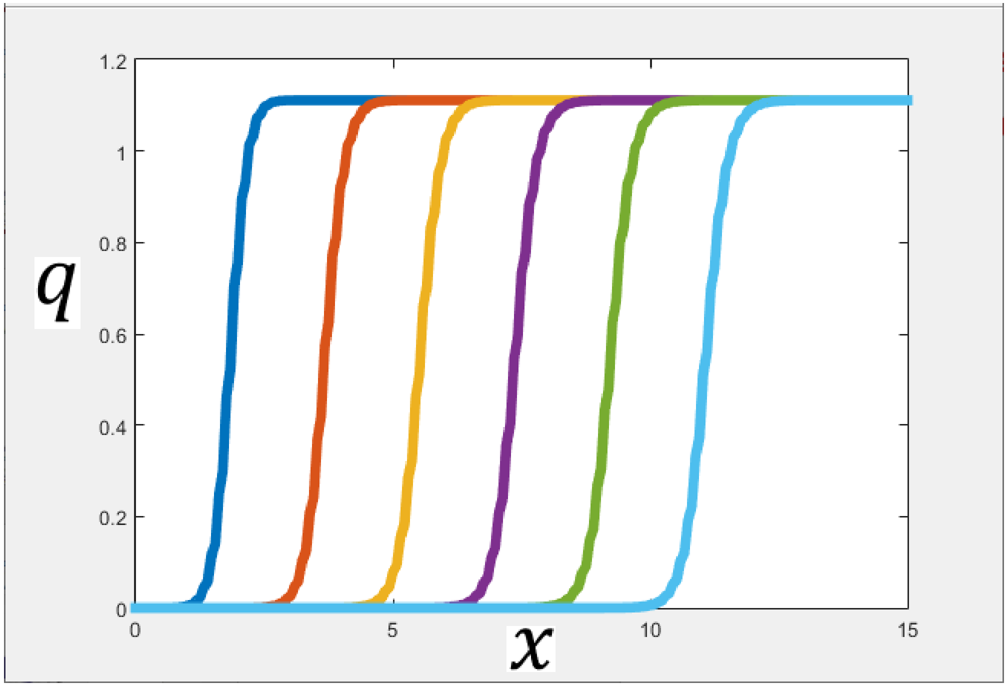
Numerical simulation of the LWR's model (16)

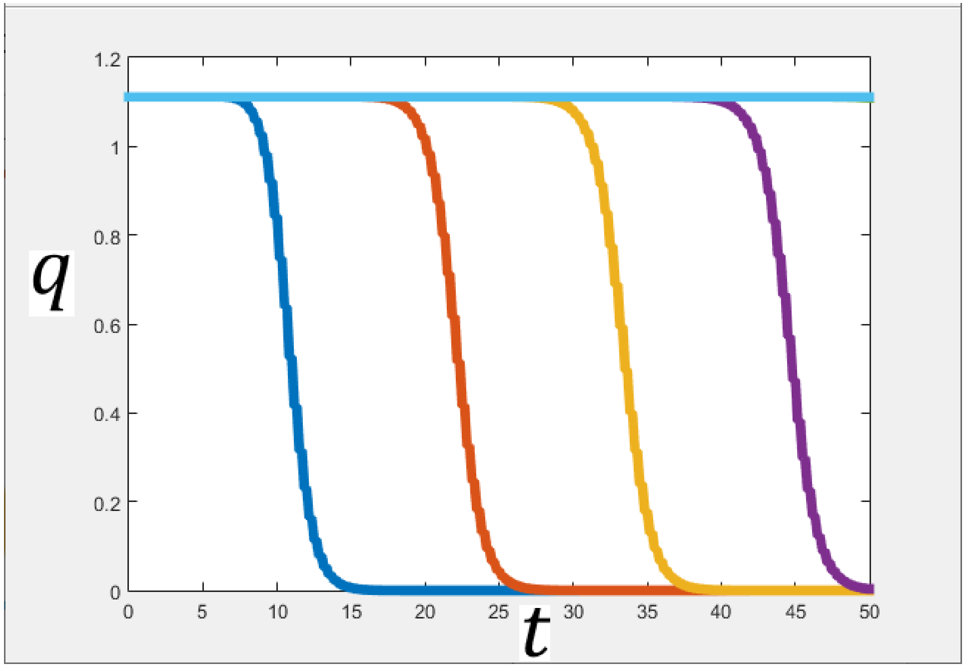
- ❑ **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 5:** Spatial evolution of the <u>flow</u> (at fixed time- slots) . → Fig. 5



- ❑ **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- ❑ **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (17)

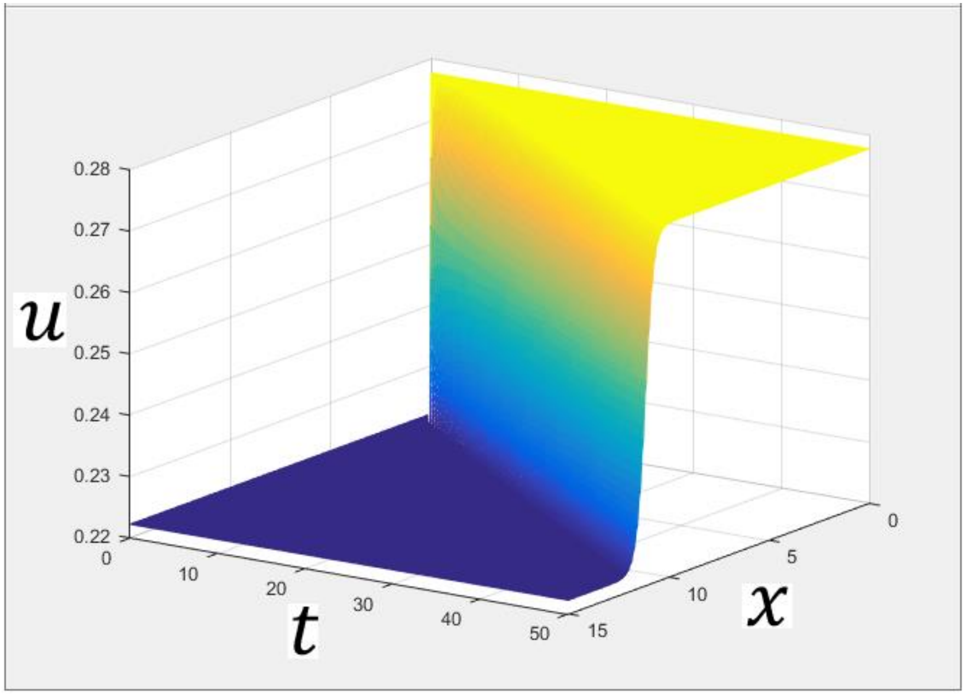
- ❑ **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 6:** Temporal evolution of the flow (at fixed positions). → Fig. 6



- ❑ **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- ❑ **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (18)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 7:** Spatiotemporal evolution of the speed . → Fig. 7



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

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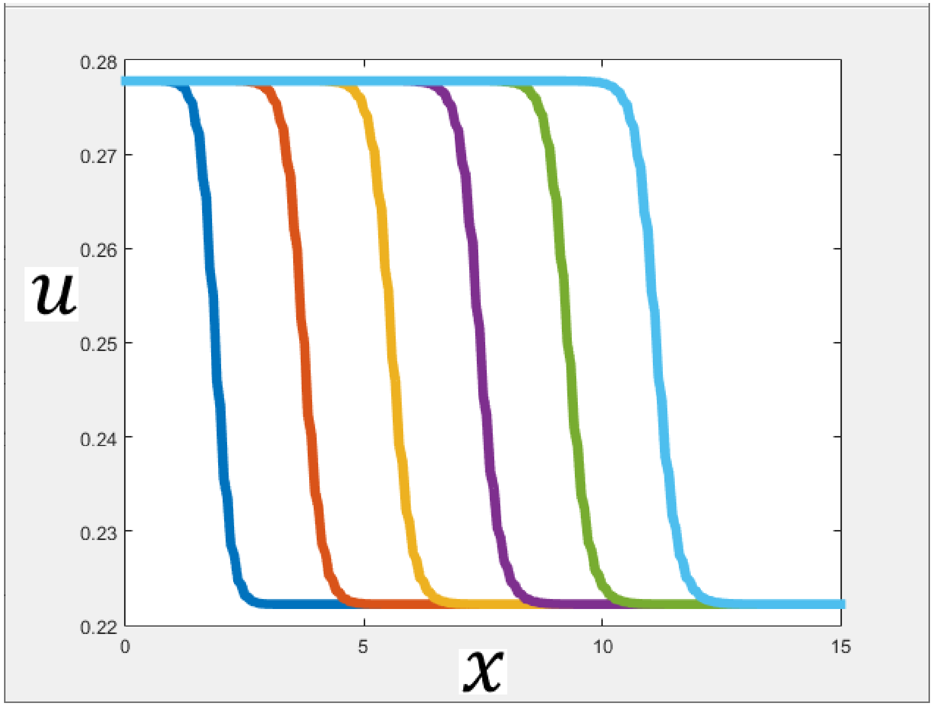
Numerical simulation of the LWR's model (19)

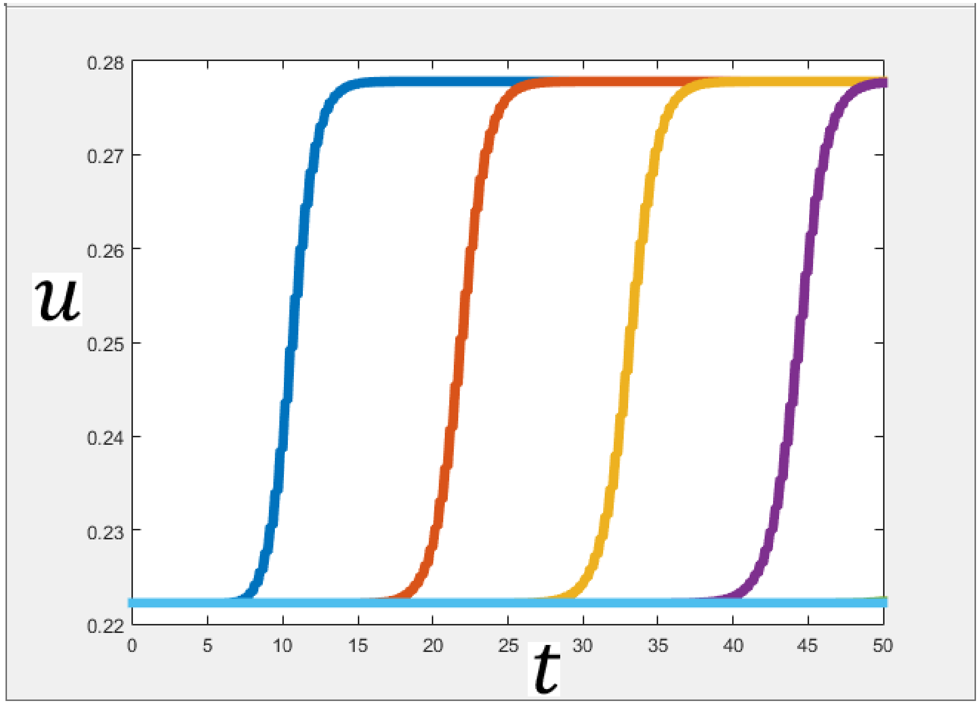
- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 8:** Spatial evolution of the speed (at fixed time- slots) . → Fig. 8



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (20)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 – Results 9** Temporal evolution of the speed (at fixed positions). → Fig. 9



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

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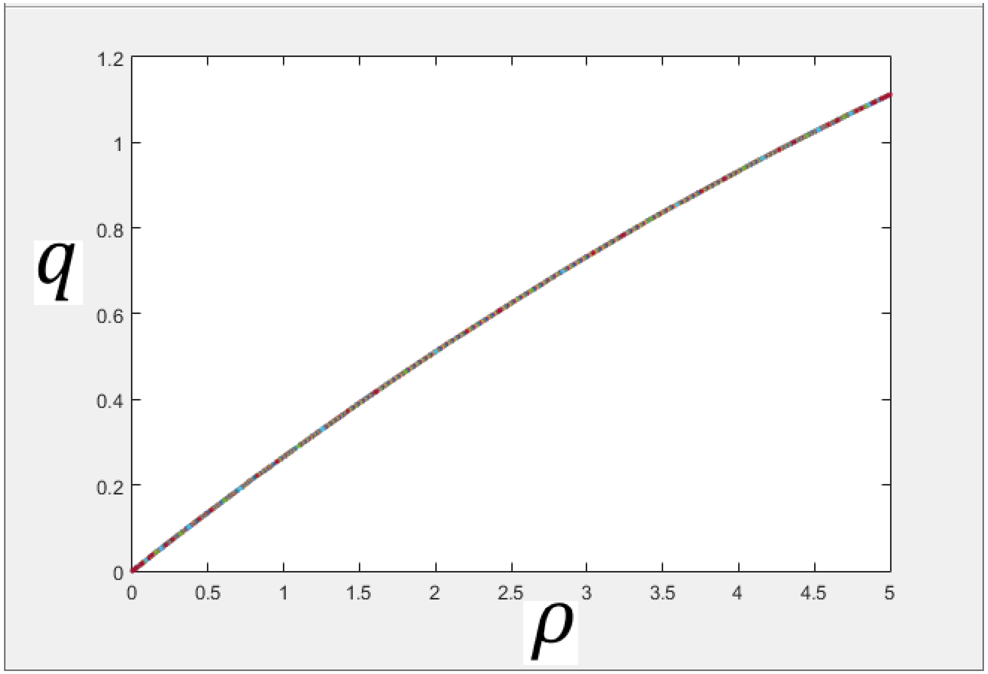
Numerical simulation of the LWR’s model (21)

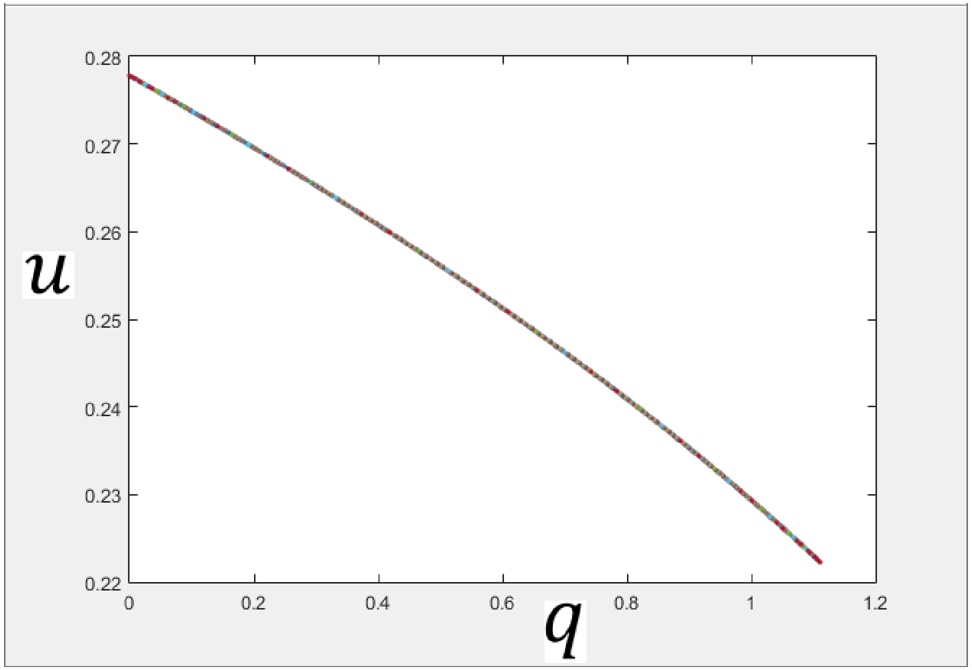
- ❑ **All steps of the Numerical solving of the LWR’s Model (14)**
  - ✓ **Step5 - Numerical results 10:** Fundamental diagram **flow** versus **density**. → Fig. 10



- ❑ **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- ❑ **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (22)

- All steps of the Numerical solving of the LWR's Model (14)
  - ✓ **Step5 - Numerical results 11:** Fundamental diagram speed versus flow. → Fig. 11



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

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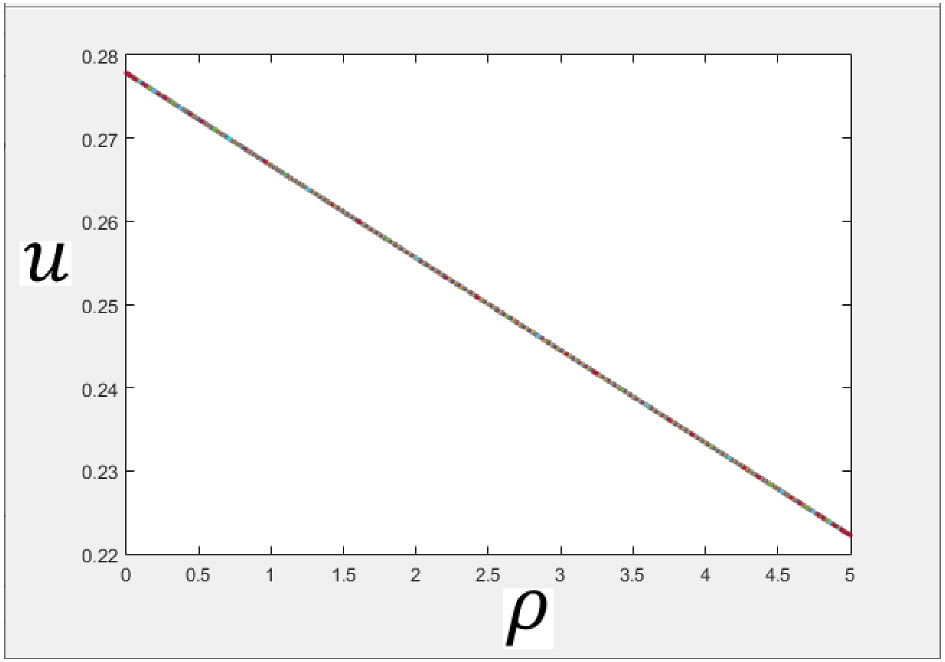
Numerical simulation of the LWR's model (23)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 12:** Fundamental diagram speed versus density. → Fig. 12



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (24)

- **CASE 2:** The parameter- settings used for the numerical simulation are defined as follows:

$$\boldsymbol{\rho(x, t_0) = 25Veh/(0.1km)}$$

$$\rho(x_0, t) = 0$$

$$\rho(x_{max}, t) = \rho(x, t_0)$$

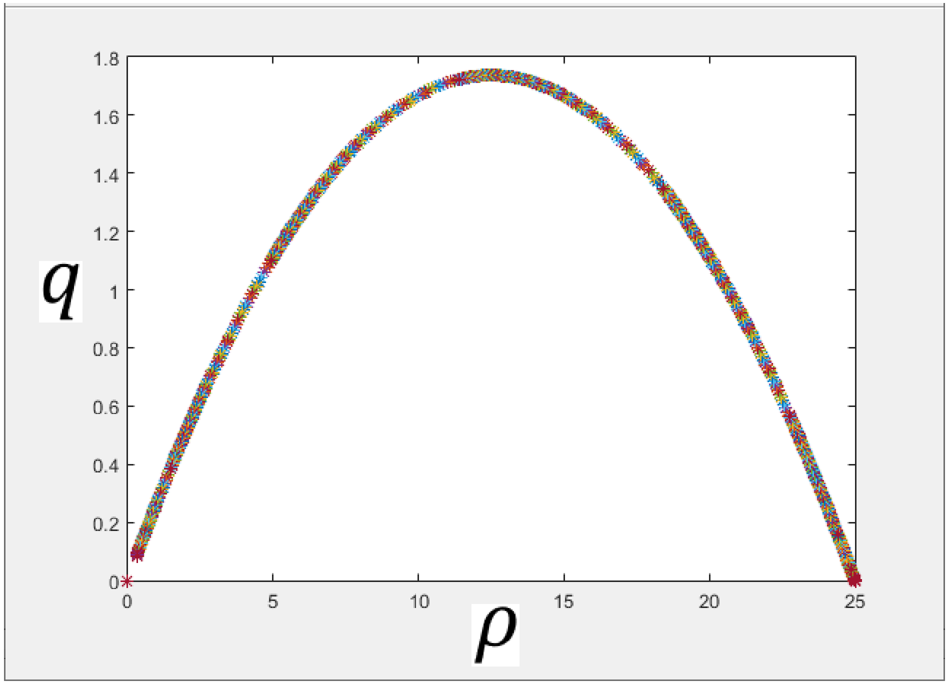
$$u_f = 100Km/h$$

$$k_j = 25Veh/(0.1km)$$

Unit of length=0.1Km=100m

Numerical simulation of the LWR's model (25)

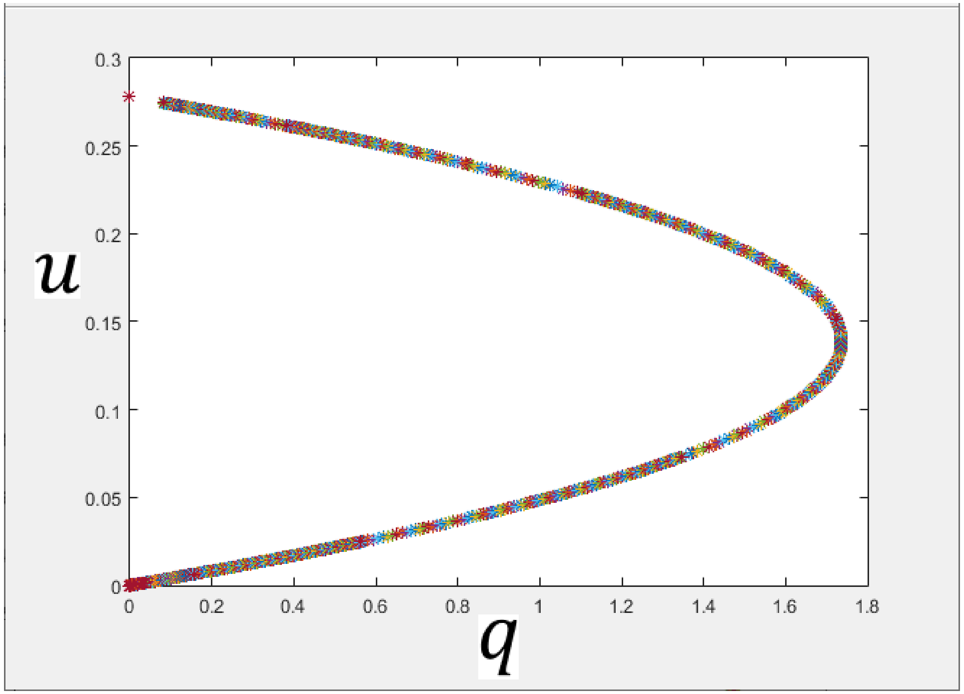
- ❑ **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 13:** Fundamental diagram <u>flow</u> versus <u>density</u>. → Fig. 13



- ❑ **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- ❑ **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (26)

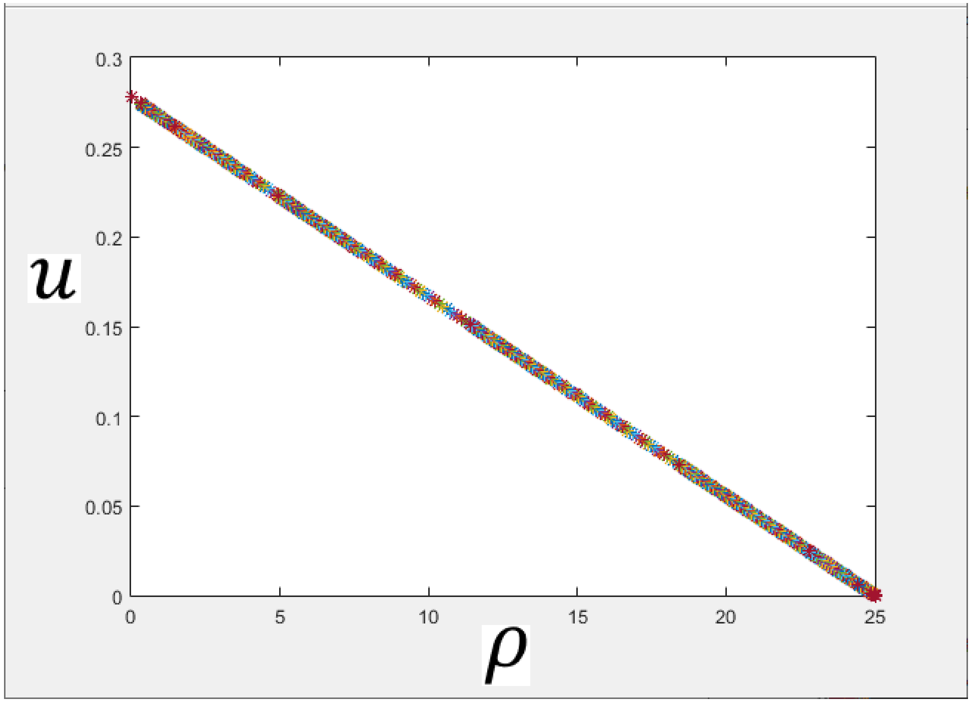
- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 14:** Fundamental diagram speed versus flow. → Fig. 14



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

Numerical simulation of the LWR's model (27)

- **All steps of the Numerical solving of the LWR's Model (14)**
  - ✓ **Step5 - Numerical results 15:** Fundamental diagram speed versus density. → Fig. 15



- **Question 1.** Comment this result in the context of the traffic scenario under investigation.
- **Question 2.** What is the meaning of this result with regards to the real-traffic on the road.

CONCLUDING REMARKS (1)

- We have described the full content of the Course/Lecture entitled „ Traffic modelling and simulation“
- We have presented some/selected concret (or real-life) scenarios requiring modelling and simulation in transportation
- We have described the important information/knowledge to be provided (in each chapter) by the Lecturer
- We have selected some projects to be considered (by the Lecturer) in the frame of this Course/Lecture.
- **As a didactic example**, we have selected amongst the nine chapters of this Course the chapter entitled **“Mathematical modelling and Numerical simulation of road traffic on Highway: Macroscopic level of details”**.

CONCLUDING REMARKS (2)

- The full content of the chapter (selected) has been presented.
- We have carried out the mathematical modeling of traffic flow on a road without ramps.
- We have shown that the traffic scenario on a road without ramps can be modeled by the LWR‘s model (expressed in the form of Partial Differential Equation (PDE)).
- We have discussed the classical methods for solving PDEs
- We have written a MATLAB CODE for solving the LWR‘s Model.
- Several plots/figures have been obtained as numerical solutions of the LWR model. These results have been commented.
- The comment of results obained has revealed various interesting traffic states (e.g. states undersaturation and oversaturation, saturation states, complete jam, shockwaves propagation, etc.).

END


Thank You


Q!