INTRAS

Co-funded by the Erasmus+ Programme of the European Union

“Mathematical methods in transportation”

Syllabus

Prof. Kyandoghere Kyamakya

Prof. Jean Chamberlain Chedjou

Intelligent Transport Systems: New ICT – based Master’s Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

GENERAL OVERVIEW OF THE LECTURE

- Efficient transportation and logistics are crucial to modern society. In fact, some key issues to be investigated (in transportation and logistics) are the production management, the resolution of a company logistical problems and the organization of the supply chain to and from the company. These issues amongst many others necessarily involve road and railway transportation. This lecture aims to train students to give them technical knowledge of traffic, transportation and logistics. The focus is on the modeling and optimization techniques in transportation. Models are derived to explain and control the dynamics of some concrete selected systems/scenarios in transportation and logistics.
- The following aspects are investigated:
  - ✓ Methods and models in transportation
  - ✓ Traffic and transport (Road and/or Railway)
  - ✓ Supply chains and logistics
  - ✓ Scheduling
  - ✓ Forecasting

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Chapter 1.
General Introduction

(Full chapter presentation)

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

1.1. Mathematics as important instrument for improving traffic and transport

Improving traffic and transport using mathematics is an old dream (1900‘s), which is nowadays effective in many disciplines (or fields of Science) through the use of models, concepts, algorithms, and tools like:

Operations research (1960s)

- In Railways
- In Roads
- In Supply Chains
- In Logistics
- Etc.

Forecasting/Prediction

- Data
- Lifetime
- Performances
- Etc.

Mathematical models

- ODEs
- PDEs
- ODES and PDEs

Optimization algorithms

- Road traffic
- Railway traffic
- Supply Chains

1.2. Definition of some keywords and concepts in transportation

- Method
- Transportation
- Traffic
- Informatics
- Logistics
- System
- Dynamic system
- System Dynamics
- Systems-theory
- Systems-identification

1.2. Definition of some keywords and concepts in transportation

What is a Method?

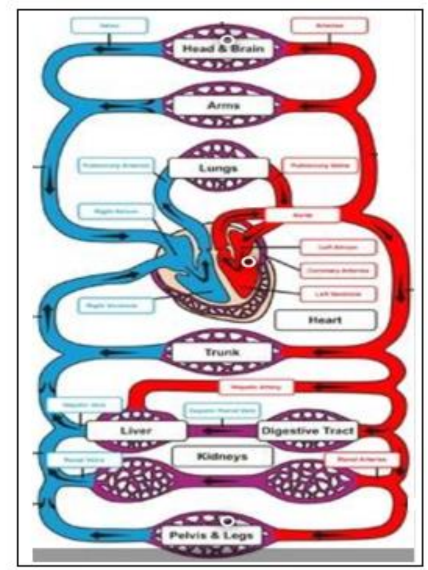
- **Method (General View):** How to do or make something
- **Method (in science):** A series of steps taken to acquire knowledge
- **Method (in computer science):** A piece of code associated with a class or object to perform a task
- **Examples:**
  - ✓ Methods for solving equations
  - ✓ Methods for management
  - ✓ Methods for traffic control
  - ✓ Methods for traffic planning
  - ✓ Methods for drawing/design
  - ✓ Etc.

Co-funded by the
Erasmus+ Programme
of the European Union

1.2. Definition of some keywords and concepts in transportation

What is Transportation?

- **Transport (General View):** (Re-) Distribution of matter or other properties of space (e.g. A scalar such as: Mass or energy, A vector such as: Momentum) in space and time.
- **Transportation (General View):** Movement of entities (e.g. People, Goods, Matter, Information/Messages, Energy, Sounds,......) from one place/point to another.



Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

Co-funded by the
Erasmus+ Programme
of the European Union

1.2. Definition of some keywords and concepts in transportation

What is Informatics?

- **Informatics (General View):**
  - ✓ Science of information (…the concept of information is closely related to notions of constraint, communication, control, data, instruction, knowledge, meaning, mental stimulus, pattern, perception, representation…),
  - ✓ Practice of information processing,
  - ✓ Engineering of information systems (system of persons, data records,…).

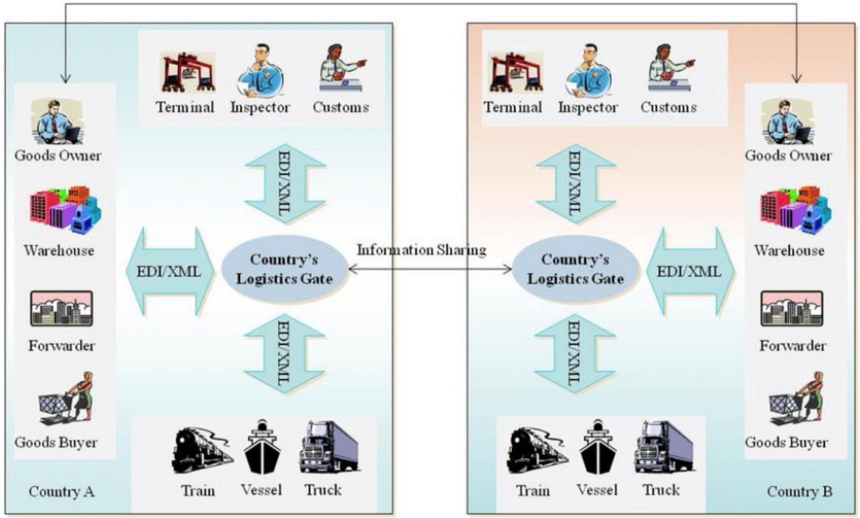
- **Informatics** studies the structure, algorithms, behavior, and interactions of natural and artificial systems that store, process, access and communicate information.

- **Informatics** develops its own conceptual and theoretical foundations and utilizes foundations developed in other fields

1.2. Definition of some keywords and concepts in transportation

What is Logistics?

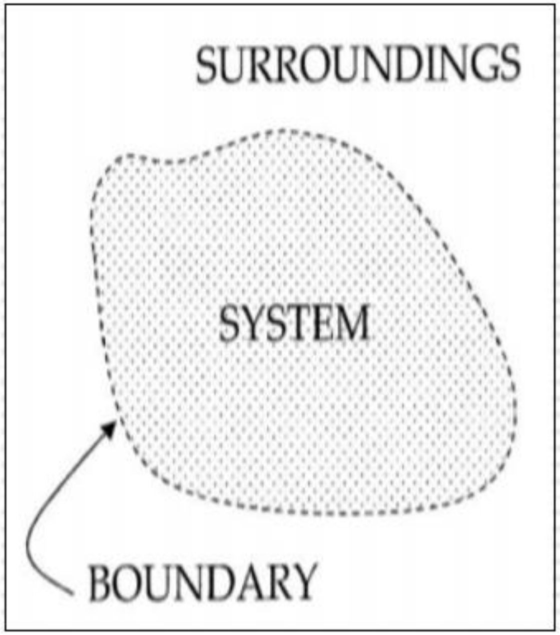
- **Logistics (General View):** Management of the flow of entities (e.g. goods, information, energy, people,...) between two points in order to meet some specific requirements.
- **Logistics involves:** * The integration of information; * Transportation, * Inventory, * Packaging,...



1.2. Definition of some keywords and concepts in transportation

What is a System?

- **System (General View):** A set of interacting, interrelated, or interdependent elements/entities (real or abstract) forming an integrated or complex whole.



1.2. Definition of some keywords and concepts in transportation

What is a dynamic System?

- **Dynamic System (General View):** A system which behavior/dynamics evolves over time.

1.3. Principles of modeling Dynamic Systems in transportation

- What is Modeling?
- Process of Modeling
- What is System identification?
- What are the advantages of Modeling in Engineering?
- What are the problems/limitations faced by the modeling process of Engineering systems?
- Sample Models of few Engineering Systems

1.3. Principles of modeling Dynamic Systems in transportation

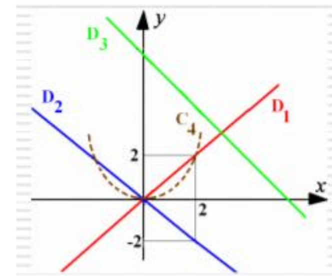
What is modeling?

- **Modeling (General View):** A process of generating abstract, conceptual, graphical, and/or mathematical, models.
- **A Model is** a Physical, mathematical, graphical, or logical representation of a system entity, phenomenon, or process.
- **A variety of things** are commonly referred to as models: physical objects, fictional objects, set-theoretic structures, descriptions, equations, or combinations of some of these.
- **Two models of the same phenomenon** may be different (principle of conceptualization)

$$y = ax^2$$

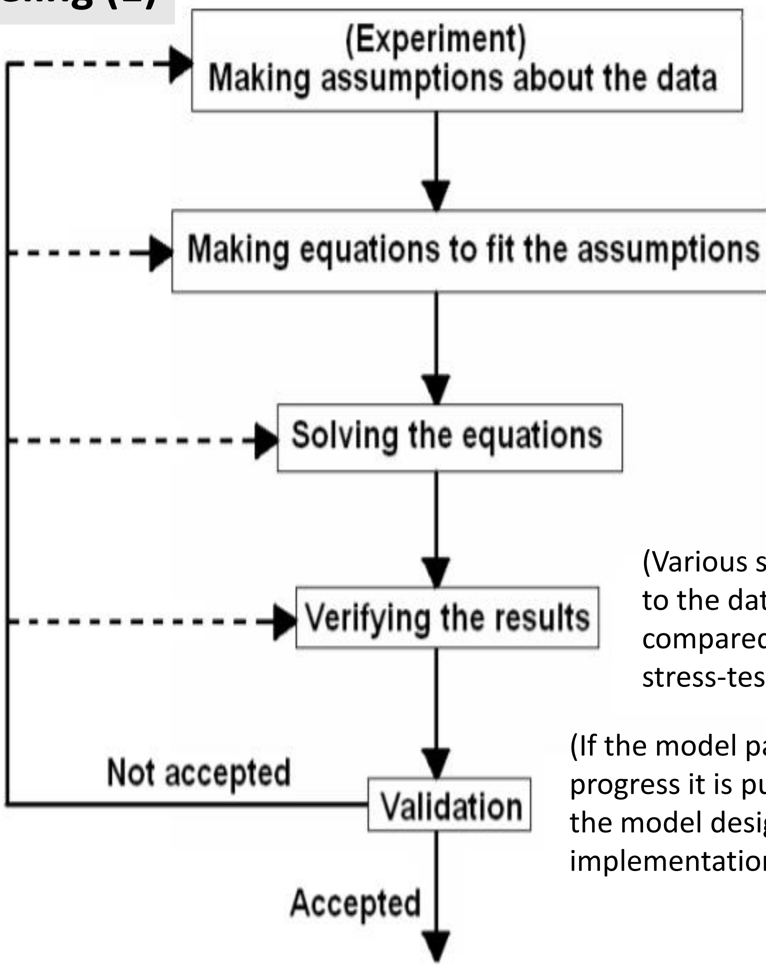
$$y = ax + b$$

Binary: 1001_2



1.3. Principles of modeling Dynamic Systems in transportation

Process of Modeling (1)



(The modeller decides what is influencing (or what affects) the data and what can safely be ignored)

(System Identification)

(Various statistical tests are applied to the data and the model is compared). This corresponds to the stress-testing of the model.

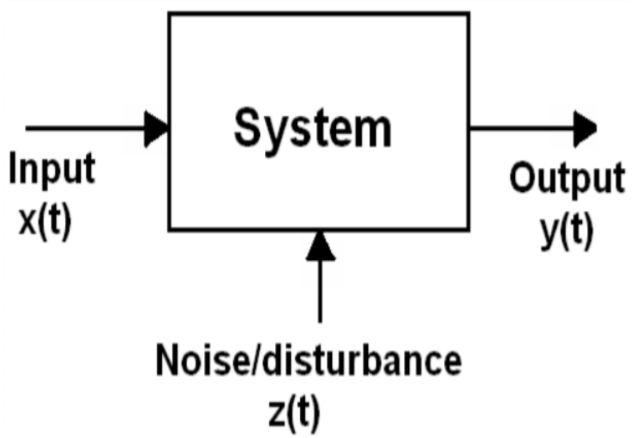
(If the model passes the verification progress it is put in practice. Therefore the model designed is ready for implementation).

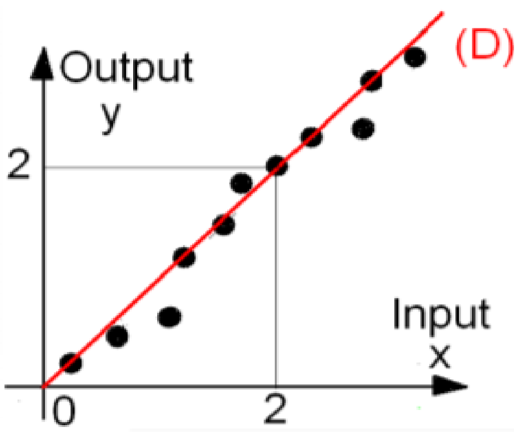
- **System identification** is the key step of the modeling process

1.3. Principles of modeling Dynamic Systems in transportation

Process of Modeling (2)

- **Experiment:** The purpose of the experiment is to collect a set of data that describes how the system behaves over its entire range of operation. The idea is to vary the input(s) and observe the impact on the output(s).





1.3. Principles of modeling Dynamic Systems in transportation

What is system identification?

- **System Identification (General View):** The task of inferring (i.e. deducing) a mathematical description (i.e. a mathematical model) of a dynamic system from a series of measurements performed on the system.

- **Different ways to approach the identification problem:**
  - ✓ **Identification process as Black-box modelling:** This implies no knowledge about the physics of the system.
  - ✓ **Identification process as White-box modelling:** This implies good knowledge about the physics of the system.
  - ✓ **Identification process as Gray-box modelling:** This implies existence of a certain level of insight about the system and exploitation of this to improve the empirical modeling.

Co-funded by the
Erasmus+ Programme
of the European Union

1.4. Examples of systems' models in transportation

How to apply system identification?

- **Below are few systems at stake:**
  - ✓ Are the systems below dynamic of static?. Justify your answer.
  - ✓ Discuss the levels of modeling and/or the modeling strategies to be considered
  - ✓ Define/Propose models for each of the systems at stake.
  - ✓ Discuss the importance of the identification process in the modeling procedure of the systems at stake.
  - ✓ Considering the systems at stake, which of these types of modelling are needed?: Black-box modelling, White-box modelling, and Gray-box modelling

1.5. Mathematical modeling in transportation: Pros, Cons and related challenges

What are the advantages of modeling?

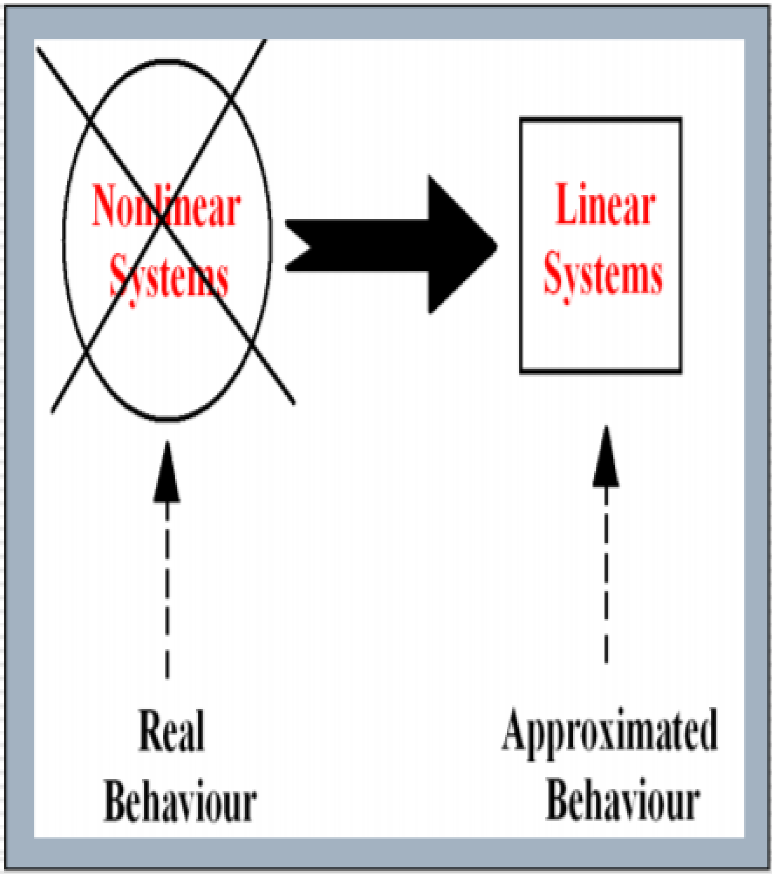
- **Advantages of Modeling (General View):**
  - ✓ Ease control/analysis of the system
  - ✓ Ease investigation of bifurcation issues
  - ✓ Allows the study of the simulated behavior of a system without having the real system. The access to real systems may be very difficult because they are costly or could be difficult to found or to realize.
  - ✓ Less Costly (Financial purposes)
  - ✓ Good security for the Modeler

- **Educational aspect of modeling:**
  - ✓ The modeler learns how to view the natural system under study in its entire complexity and then how to reduce the system to the relevant processes and the relevant parameters.

1.5. Mathematical modeling in transportation: Pros, Cons and related challenges

What are the limitations/drawbacks of modeling?

- Limits of the Modeling process (General View):
  - ✓ Real systems are nonlinear.
  - ✓ Nonlinear models to describe real systems are very difficult to propose/obtain (due to the identification process).
  - ✓ Exact analytical solutions of nonlinear models are impossible (only approximate solutions exist)
  - ✓ Linear models to describe nonlinear systems is the most common approach in engineering. This approach cannot give full insights of the real system. Some interesting features such as shockwave, rarefaction wave, torus, chaos, etc.. may not be detected by the mathematical models obtained as results of the modeling process.

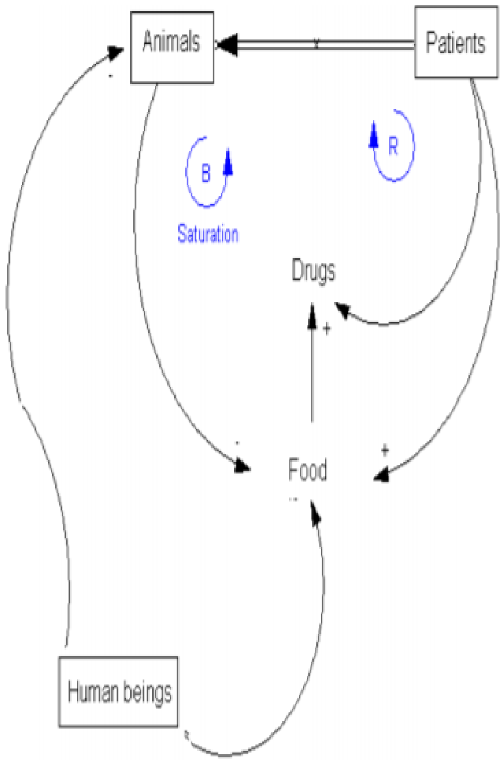


1.6. Fundamentals of system theory

What is System Theory?

- **System theory (General View):** A framework by which one can analyze and/or describe any group of objects (e.g. A single organism, an organization, a society, an electromechanical system, Supply chain networks, Coupled/cooperating machines...) that work in concert to produce some result.
- There exist models, principle, and laws that apply to generalized systems or their subclasses.
- Aim of general system theory
  - ✓ Establish general system laws which apply to any system of a particular type.
- These considerations lead to the postulate of a new scientific discipline which we call general system theory.
- General theory of systems would be a useful tool for providing/building on models that can be used in different fields.

1.6. Fundamentals of system theory

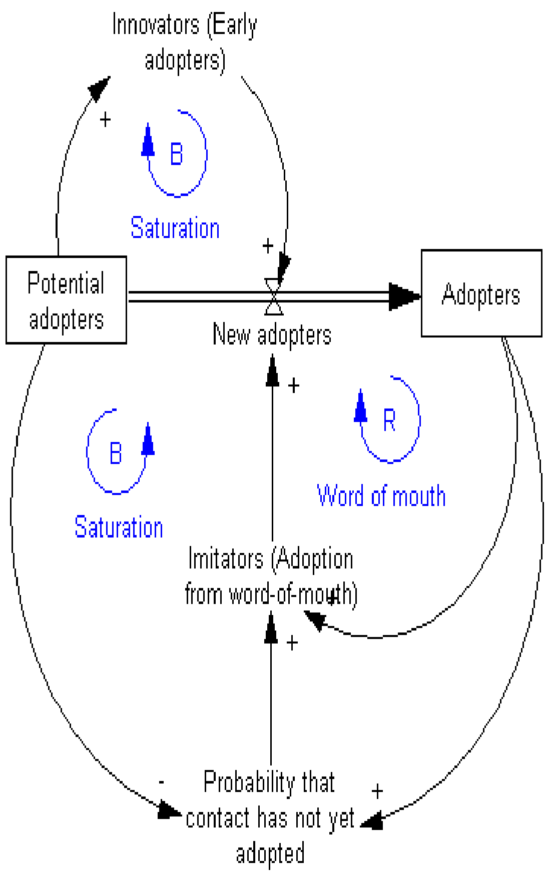


Systems dynamics modeling, [Ref. John Sterman 2001]

What is System Dynamics?

- **System Dynamics (General View):** An approach used to represent, analyze and understand the behavior of complex systems over time.

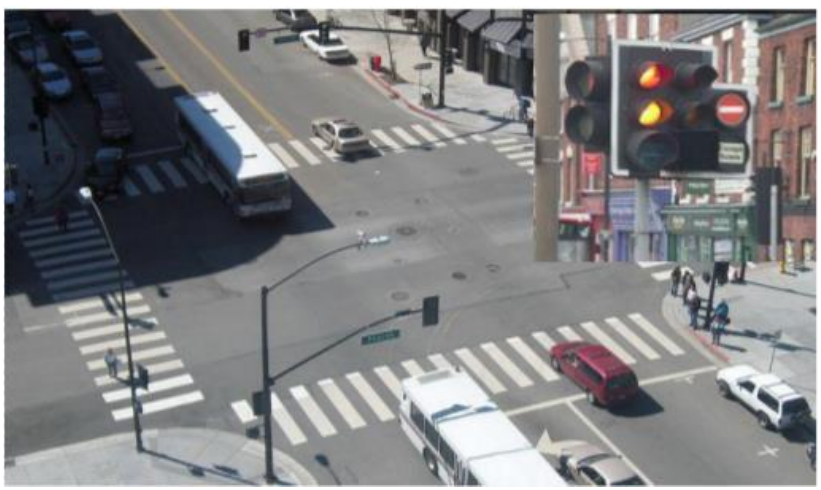
- **Exercise:** Provide/give a full description of the two graphical models on this slide.

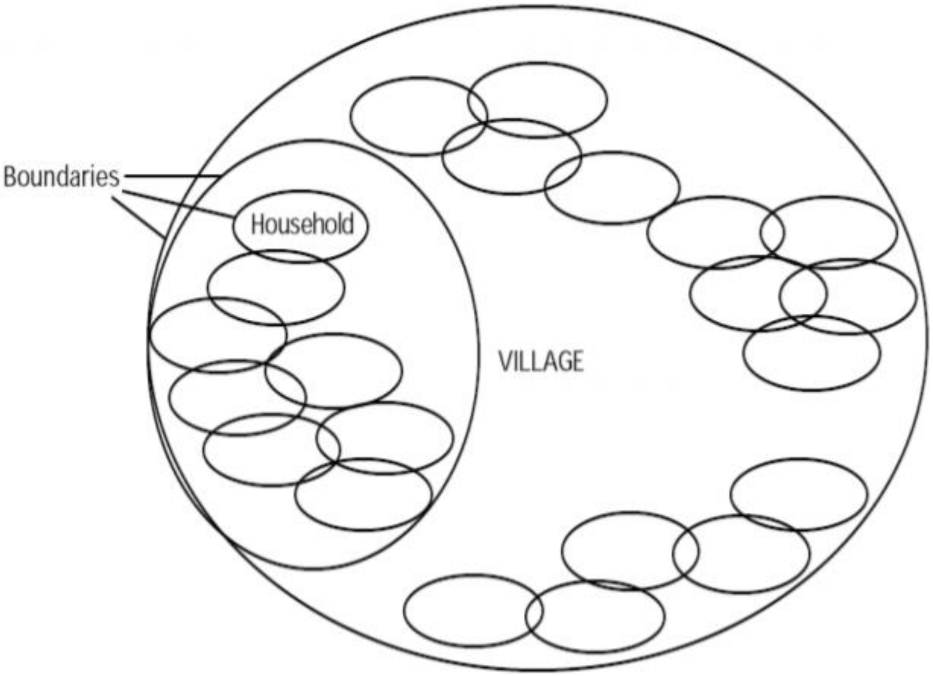


1.6. Fundamentals of system theory

General/Global view of a System

- **System** (Macroscopic view)
- **Sub-Systems** (Partial/Restricted view)
- **Boundaries** (Partial/Restricted view)
- **Entities** (Microscopic view)





1.7. Evaluation of modeling techniques in Transportation

Two main groups of Models exist for modeling transportation systems:

- **Group 1. Abstract mathematatic models.**
  - ✓ Ordinary differential equations ODE (Microscopic traffic)
  - ✓ Partial differential equations PDE (Macroscopic traffic)
  - ✓ Mixture of ODE and PDE (Mesoscopic traffic)
  - ✓ Time series models (for traffic demand forecasting)
  - ✓ Averaging and smoothing models (for the calculation of average and/or exponentially smoothed prediction)
  - ✓ Regression analysis (for identifying the mathematical relationship which best fits the data at stake).

1.7. Evaluation of modeling techniques in Transportation

Two main groups of Models exist for modeling transportation systems:

- **Group 1. Abstract mathematic models.**
  - ✓ Regression analysis (can also be used to predict metrices such as :
    * waiting times at traffic junction, * numbers of accidents,
    * Rate/levels of pollution, etc..
  - ✓ Matrix estimation models (for estimating the trips in "origin to destination" pairs).
  - ✓ Elasticity models (for measuring the effects of changes of traffic conditions on the traffic demand forecast)
  - ✓ Demand allocation – modelling the choice between alternatives (for integrating constraints related to the choice of specific modes of transportation, start-time and trip duration, route choice w.r.t congestion, energy consumption, pollution, etc.).

1.7. Evaluation of modeling techniques in Transportation

Two main groups of Models exist for modeling transportation systems:

- **Group 2. Simulation models (also called traffic simulation tools).**
  - ✓ Simulation models are built using very complex mathematical formulas (Black box modeling). This is the main similarity compared to the abstract models in Group 1.
  - ✓ Compared to abstract models (in Group 1), simulation models can integrate (or consider) the explicit representation of the behavior/dynamics of traffic stakeholders (e.g. types of cars, speeds of cars, roads and intersections geometries, traffic signals timings, etc.)
  - ✓ Simulation models can allow scientists/researchers to develop a logic model of a very complex system even without having a theoretical background on system dynamics and also without a solid background in the field of Dynamic systems.

1.7. Evaluation of modeling techniques in Transportation

Two main groups of Models exist for modeling transportation systems:

- **Group 2. Simulation models (also called traffic simulation tools) are classified into three categories:**
  - ✓ **Macroscopic models:** Simulate traffic flow dynamics/behavior using average aggregate traffic stream characteristics, such as flow rate, speed and traffic density.
  - ✓ **Mesoscopic models:** Simulate the movement of individual cars/vehicles using simplified car/vehicle behavioral models, often based on macroscopic principles. The driver behavior is another important issue investigated by mesoscopic models.
  - ✓ **Microscopic models:** Simulate the movement of individual cars/vehicles through the application of driver behavioral models.

1.8. Projects: Sample illustration of real-life problems in Railway transportation

- SAPI: Statistical Analysis of Propagation of Incidents
- Mathematical approach applied to train scheduling
- Mathematical modeling of Railway Rescheduling Problem
- Mathematical model of wheel pairs movement of a rail vehicles
- Mathematical modeling, optimization and parameters computations in multimodal freight terminal networks
- Multicommodity Network Design Problem in Rail Freight Transportation Planning
- Mathematical and simulation techniques for modeling urban train networks
- A mathematical model of selecting transport facilities for multimodal freight transportation
- Mathematical model of optimal empty rail car distribution at railway transport nodes
- Operations research in passenger railway transportation
- A Mathematical Model for Railway Control Systems - (NASA Contractor Report 198353)

1.8. Projects: Sample illustration of real-life problems in Railway transportation

- ❑ Mathematical model for planning the distribution of locomotives to meet the demand for making up trains
- ❑ A mathematical model of the rail track presented as a bar on elastic and dissipative supports under the influence of moving loads
- ❑ A mathematical model to study railway track dynamics for the prediction of vibration levels generated by rail vehicles
- ❑ Mathematical models and methods for analyzing computer control networks of railway power supply
- ❑ Mathematical Optimization of rail station location and route design in urban regions through minimizing noise pollution
- ❑ A Mathematical Model of Railway ROW Soil Cleanup Using Sorbent
- ❑ Mathematical modelling of a diesel common-rail system
- ❑ Economic and Mathematical Model for Forecasting Passenger Traffic on a Long-Term Basis Case of Study Russia
- ❑ Development of a Novel Freight Railcar Load Planning and Monitoring System

1.8. Projects: Sample illustration of real-life problems in Road transportation

- A Road Traffic "behavioral analysis": Mathematical modeling and Numerical Simulation
- Mathematical modelling of traffic flow on highway
- Mathematical modeling of road transport in context of critical infrastructure protection
- Mathematical models for traffic control with concrete applications
- On Mathematical model for the study of traffic flow on highways
- A Neural Network Model for Drivers Lane-Changing Trajectory Prediction in Urban Traffic Flow
- Simulation research on chaos characteristics of a class of macroscopic traffic models
- A Multi-commodity Lighthill-Whitham-Richards Model of Lane-changing Traffic Flow
- A new continuum model for traffic flow and numerical tests
- A new anisotropic continuum model for traffic flow
- Feedback Ramp Metering using Godunov method-based hybrid model

1.8. Projects: Sample illustration of real-life problems in Road transportation

- Exploring the Behavior of LWR Continuum Models of Traffic Flow in Presence of Shock Waves
- Minimizing Carbon Emissions from Transportation Projects in Sub-saharan Africa Cities Using Mathematical Model
- Mathematical Modelling Using Integer Linear Programming Approach for a Truck Rental Problem
- A Mathematical Model in Reduction of Cost on Transportation of Sugarcane and the Loss Due To the Accident in Transportation
- Formulating a mathematical model for container assignment optimization on an intermodal network
- Modeling nonlinear road traffic networks for junction control
- Traffic signals control in a network of coupled junctions using Kuramoto model
- Dynamic modeling study on the slope steering performance of articulated tracked vehicles
- Research on Driver Behavior in Yellow Interval at Signalized Intersections
- Bayesian analysis of traffic flow on interstate I-55: The LWR model

1.8. Projects: Sample illustration of real-life problems in Supply chains networks & logistics

- Modelling Supply Chains with PDEs
- Supply Chain Evidential data mining and stochastic
- Modelling of a Supply Chain Network Driven By Stochastic Fluctuations
- Modelling of Job Shop Scheduling Problem
- Supply Chain Management Optimization Problem
- Supply Chain Network Design under Uncertainty with Evidence Theory
- Optimization in supply chains (Transportation assignments)
- Application of cost models in transportation companies
- Modelling and optimization of the assignment problem in a supply chain network
- Modeling and optimization of the Crew scheduling problem in Railway transportation
- Scheduling Additional Train Unit Services on Rail Transit Lines
- Bi-Objective Modelling for Hazardous Materials Road–Rail Multimodal Routing Problem with Railway Schedule-Based Space–Time Constraints

Co-funded by the
Erasmus+ Programme
of the European Union

1.9. Transportation systems/scenarios undergoing nonlinear and time varying dynamics

Rail Traffic
(Dynamics of Trains)

Road Traffic
(Dynamics of Vehicles)

Supply chain Networks (SCN)
(Dynamics of Goods)

Adaptive traffic control systems
(Actuated control systems)

Supply Chain Management
(Information Exchange)

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Chapter 2.

Basics of graph theory and applications in transportation

(Full chapter presentation)

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

2.1. Sample applications of graph theory in transportation

- 2.1.1. Case of a telecommunication scheme (V2V & V2I)
  - ✓ Terminals (equipments interfaced with end users)
  - ✓ Network equipments
    - Active (e.g. Routers)
    - Passive (e.g. Reflectors)

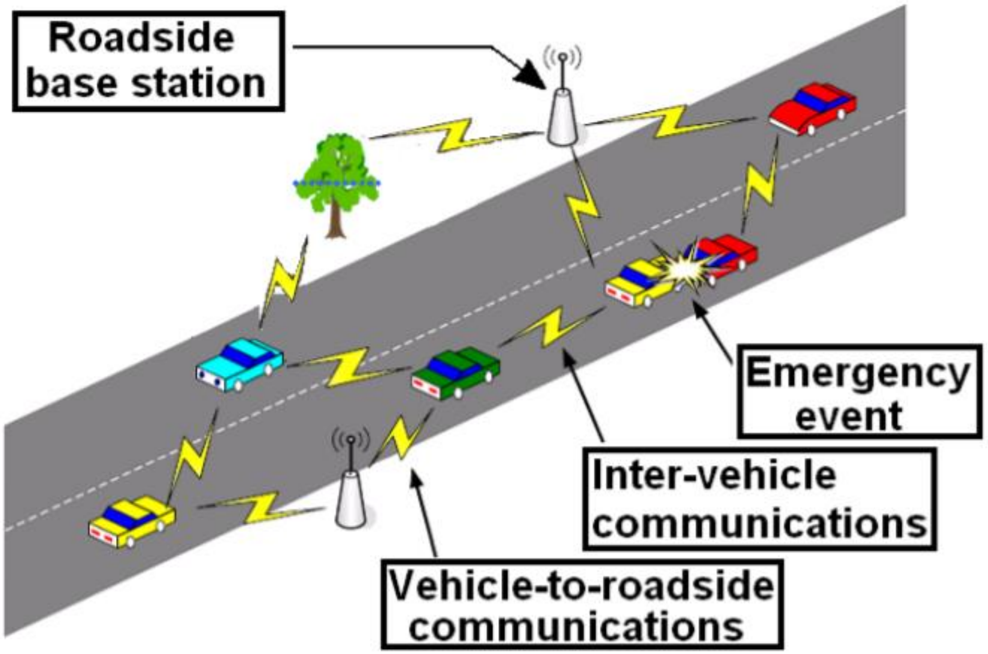


Fig. 1. Illustration of V2V & V2I communications

2.1. Sample applications of graph theory in transportation

- 2.1.2. Case of a Road traffic signals
  - ✓ Use of synchronized coupled oscillators to model and optimize Realtime self-organized traffic control in a city.
  - ✓ Adapt the analog simulation (based- CNNs) to real traffic scenarios.
  - ✓ Kuramoto model for self-organized road traffic signals control (see figures and equations on this slide).

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i), \quad i = 1, \ldots, N$$

$$\tilde{\omega}_i = \lim_{t \to \infty} \frac{1}{t} \int_0^t \dot{\theta}_i dt$$

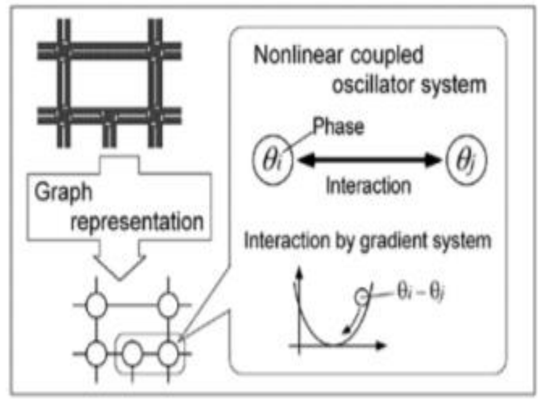


Fig. 2. Overview of the method

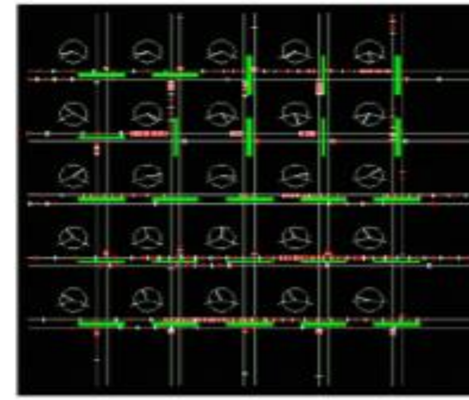


Fig. 3. Traffic simulator

Fig. 4. An isolated traffic junction

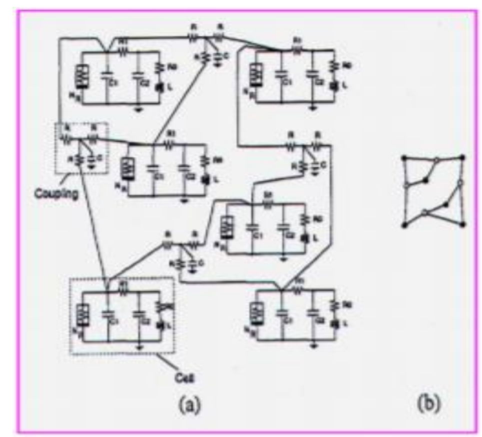


Fig. 5. Representation in form of graph

2.1. Sample applications of graph theory in transportation

- 2.1.3 Case of a Railway traffic scenarios
  - ✓ On the Use of Graph Theory for Railway Power Supply Systems Characterization (Fig. 6).
  - ✓ A graph model for the management of the motion of Trains in order to avoid conflicts (Fig. 7).

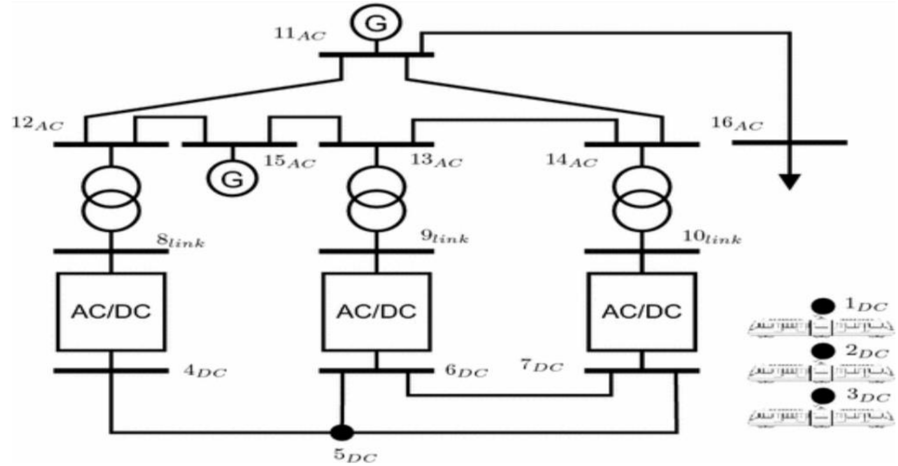


Fig 6. Graph Theory for Railway Power Supply Systems Characterization

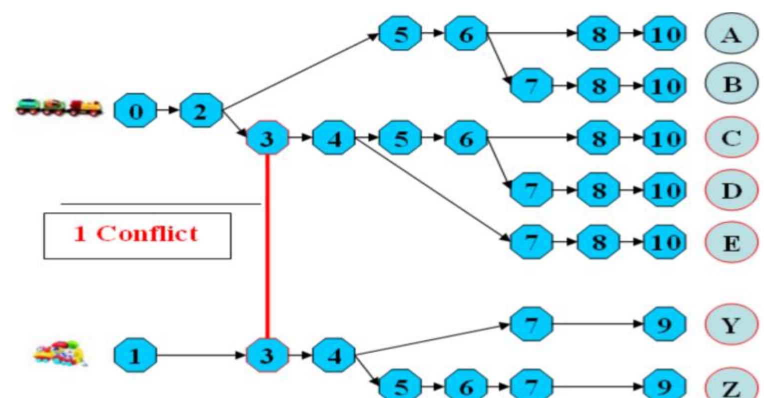


Fig 7. Graph of a Train scheduling problem

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

2.1. Sample applications of graph theory in transportation

❑ 2.1.4 Case of Supply Chain Networks (SCN)

- ✓ Graph theory can be used to develop models for optimizing supply chain distribution, and developing recommendations
- ✓ Shortest path problem (SPP) and Traveling Salesman Problem (TSP) can be used in the context of graph theory to maximize the distribution of Goods, minimize the energy consumption, pollution and time saving.

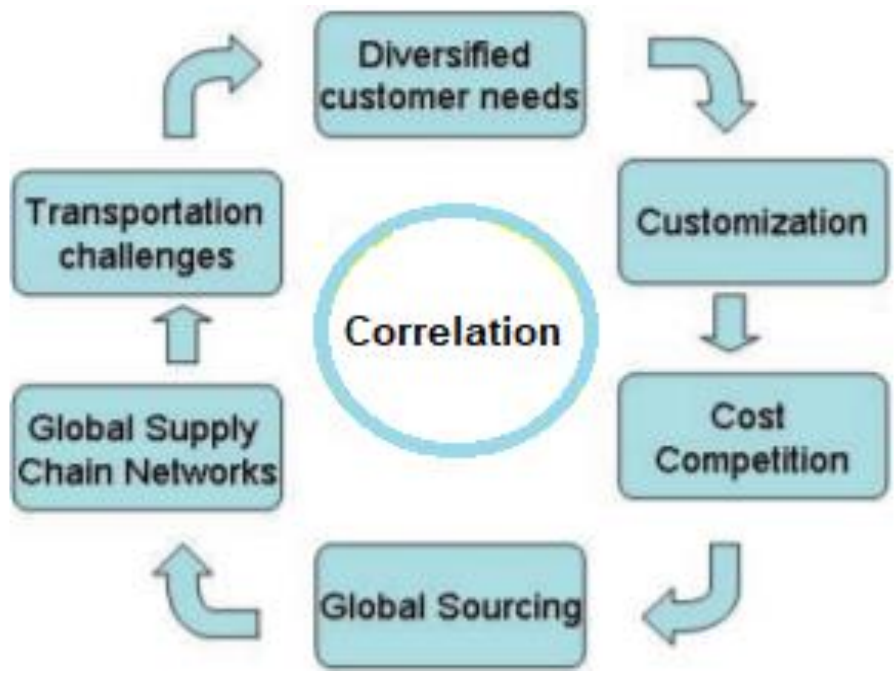


Fig 8. Complexity resulting from the correlation between sub-systems

2.2. Basic concepts in graph theory

- 2.2.1 Magnitude; Size; Edges; Weights; total cost of a graph network
  - **Magnitude.** Number of nodes/vertices of a graph
  - **Size.** Number of edges of a graph
  - **Edge/Link.** Connection between a pair of nodes
  - **Weight.** Cost of an edge
  - **Total cost of a graph.** Sum of all weights
  - **Total cost of a solution (e.g. SPP/TSP).** Sum of weights involved in the solution

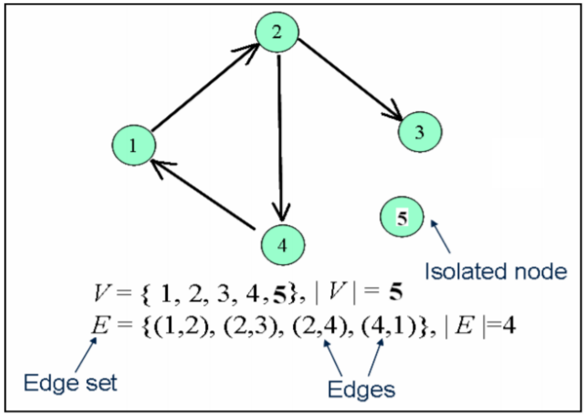


Fig 9. Scheme of a directed graph.

- 2.2.2 Directed graph; Undirected graph; Completed graph; Uncompleted graph
  - **Directed graph.** Also called digraph, each pair of nodes is connected through an edge in a specific direction revealing the source node and target node. In a directed graph edges are unidirectional. When edges are bidirectional the graph is said to be **Undirected.**
  - **Completed graph.** A simple undirected graph in which every node is connected to all other nodes. Otherwise the graph is said to be **Uncompleted**.

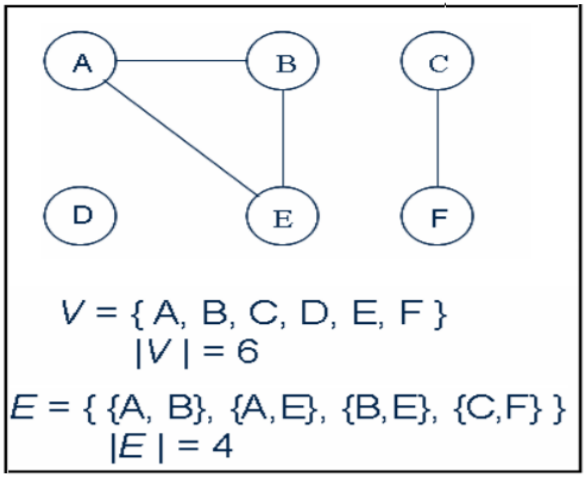


Fig 10. Scheme of an undirected graph.

2.2. Basic concepts in graph theory

- 2.2.3 Connected graph; Disconnected graph; Mixed graph;

  - ✓ **Connected graph.** A graph in which a path is established from any node to any other node. This corresponds to the fact that all nodes are reachable.
  - ✓ **Disconnected graph.** A graph in which a path is not necessarily established between all nodes. This corresponds to the fact that all nodes are not reachable (e.g., an isolated node).
  - ✓ **Mixed graph.** A graph with both directed and undirected edges.
  - ✓ **Example.** The graph in Fig. 9 is connected. This graph is said to be disconnected if the dashed edge is removed.

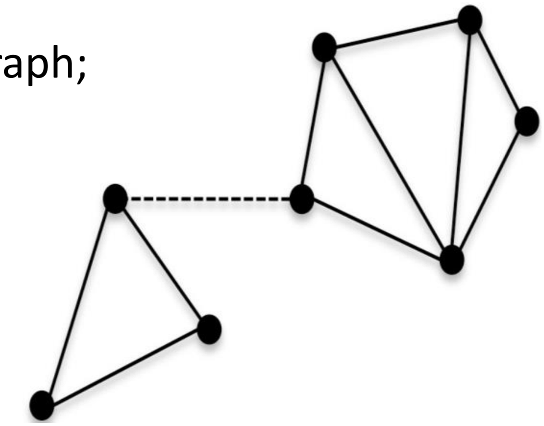


Fig 11a. Connectivity of an undirected graph network

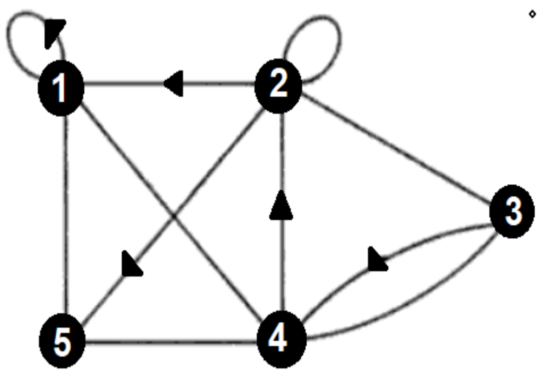


Fig 11b. A mixed graph

2.2. Basic concepts in graph theory

- 2.2.4 Graph; Real graph; Virtual graph;
  - ✓ **Graph.** A representation (graphical) of any system, phenomenon, scenario, process, etc.
  - ✓ **Elements of a graph.** Set of nodes/ vertices (V), Set of edges/lines (E).
  - ✓ **Graph notation.** G(V,E)
  - ✓ **Graph magnitude.** |V|
  - ✓ **Graph size.** |E|
  - ✓ **Nature of a Graph.**
    - *A real graph* (e.g., a graph modeling the traffic in a city);
    - *A virtual graph* (e.g., a graph modeling the traffic in Europe; e.g., An equivalent graph of a real graph)

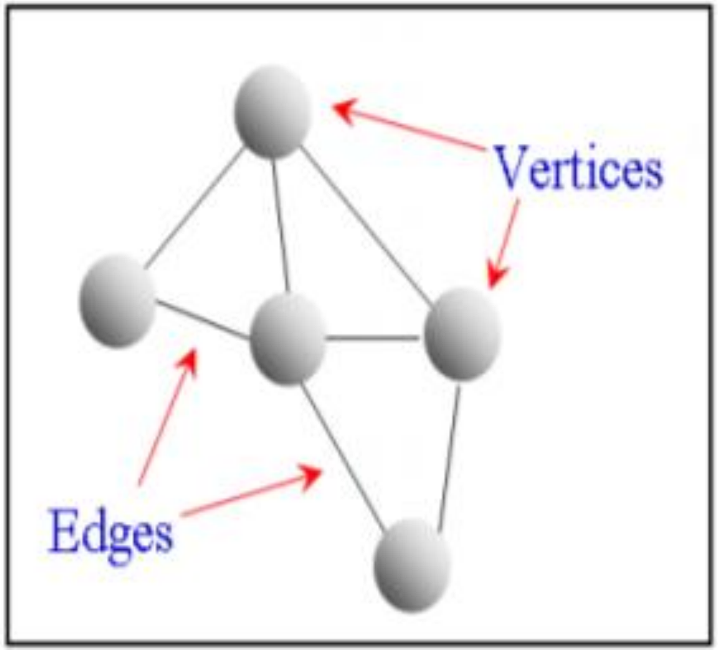


Fig 12. Representation of an undirected graph network

2.2. Basic concepts in graph theory

- 2.2.4 Graph; Real graph; Virtual graph;
  - ✓ **A graph of a telecommunication network.**
    - Nodes (Vertices) are: *Terminals (equipment interfaced with end users) and * Network equipment (Active (e.g. Routers) and Passive (e.g. Reflectors))
    - Edges (links) can be fixed by the routing protocol: (e.g., QoS, Priorities, etc.)
  - ✓ **A graph of a road traffic scenario.**
    - Nodes (Vertices) are: *Intersections (crossings/junctions); *Lanes
    - Edges (Links) are: *Distance between nodes; *Density of traffic (i.e. Degree of congestion) between nodes; *Capacity of the queue (e.g. in case lanes are considered as vertices); etc.

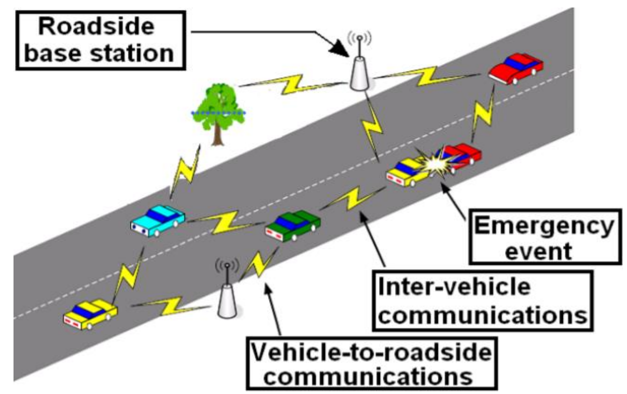


Fig 13. A telecommunication scheme (V2V & V2I).

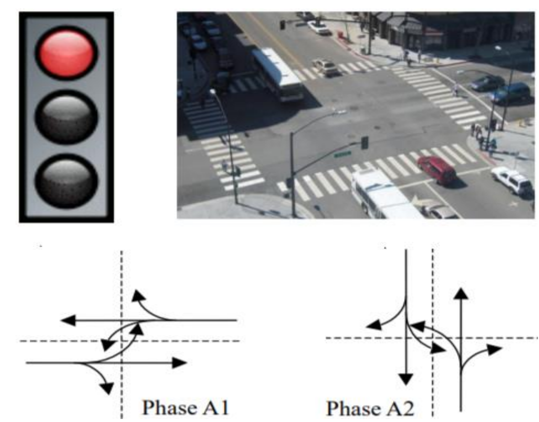


Fig 14. A traffic junction/intersection/crossing.

Co-funded by the
Erasmus+ Programme
of the European Union

2.2. Basic concepts in graph theory

- 2.2.5 Applications of graph theory
  - ✓ **A weight in graph theory.** This is a critical value between two nodes/vertices. This value is determined/defined by a routing protocol (i.e. Selection of optimized path/way).
  - ✓ **Applications of graph theory.**
    - Finding optimal/best/optimized paths (routing). Below are some concrete examples.
      - *In SCN & telecommunications:* Goods, packets/messages transmission between vertices (i.e. Transmission without loss of packets, transmission with short (or no) time delay, etc...).
      - *In road traffic scenarios:* Drivers (along a city) always look for optimal paths (i.e. Shortest paths, Paths with no (or less) congestion, etc...) from source to destination.

Fig 15. Illustration of the states of traffic flow: Undersaturated traffic, Saturated traffic and Oversaturated traffic

2.2. Basic concepts in graph theory

2.2.6 Walk; Closed walk; Trail; Path; Cycle; Loop; Forest; Length of a Walk

- **Walk.** Alternate sequence of incident vertices and edges
- **Closed walk.** The last vertex is equal to the first vertex (i.e. the last edge is adjacent to the first edge)
- **Trail.** A walk which traverses only once each edge.
- **Path.** A walk which traverses only once each vertex.
- **Cycle.** A closed path
- **Loop.** An edge connecting a vertex to itself.
- **Forest.** A graph without cycle.
- **Length of a Walk.** The number of Edges included in the walk.

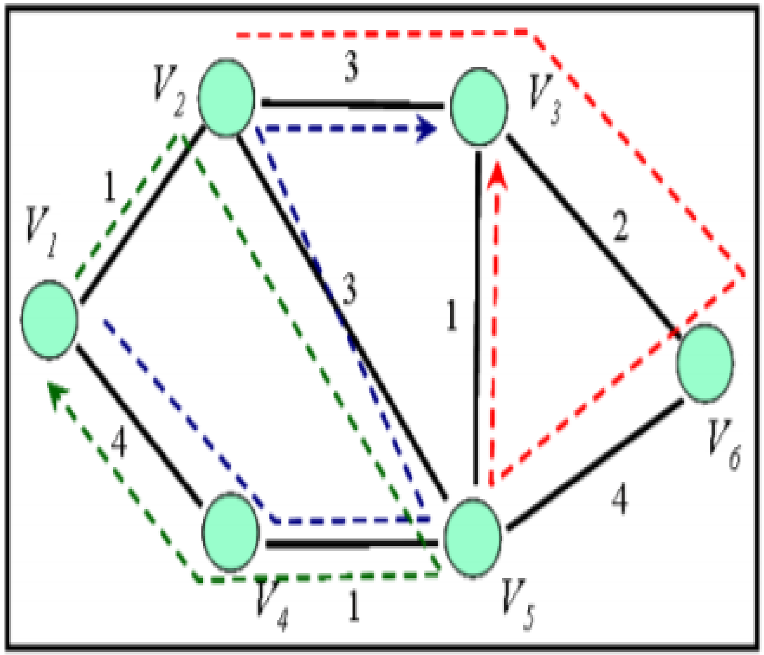


Fig 16. Illustration of a graph with Walk, closed walk, Trail, Cycle, etc.

Question: Which of the above expressions can (or cannot) be found in the graph enclosed?

2.2. Basic concepts in graph theory

- 2.2.7 Cyclic graph; Acyclic graph; Adjacent vertices; Complete graph
  - ✓ **Cyclic graph.** A graph containing at least one cycle.
  - ✓ **Acyclic graph.** A graph with no cycle.
  - ✓ **Adjacent vertices.** Any pair of vertices linked by an edge.
  - ✓ **A complete graph.**
    - An Undirected (or directed) graph in which every pair of vertices is adjacent.
    - For any n (integer) , a complete graph of n vertices (denoted by Kn) is a graph with n nodes in which every node (u) is adjacent to every other node (v) **($\forall u,v \in V: u \neq v \leftrightarrow \{u, v\} \in E$)**.

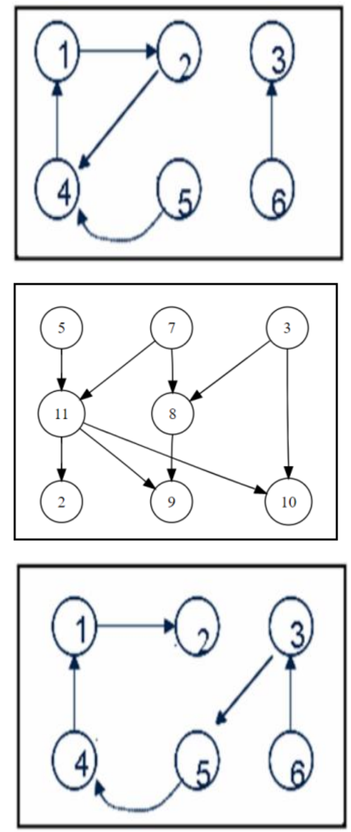


Fig 17a. Cyclic and Acyclic graphs

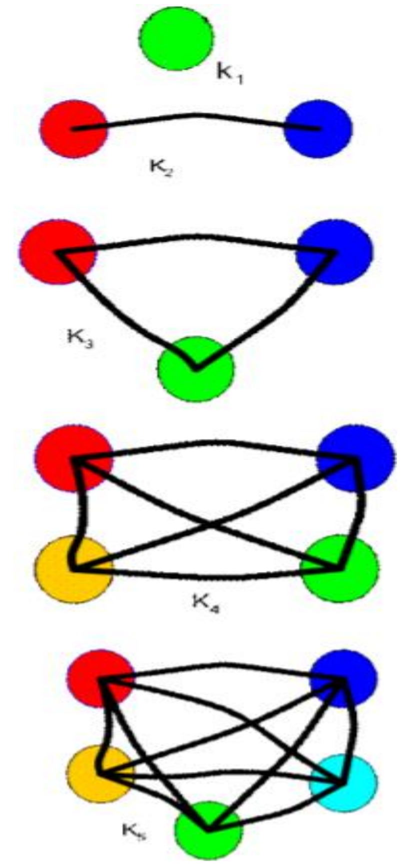


Fig 17b. Complete graphs (Kn)

- **Note that Kn has** $\sum_{i=1}^{n-1} i = \frac{n(n-1)}{2}$ **edges (case of undirected graphs)**

2.2. Basic concepts in graph theory

2.2.8 Bipartite graph

- **Bipartite graph.** An undirected graph G = (V, E) with following properties:
  - V can be partitioned into 2 sets V1 and V2 such that: (x, y) ∈ E implies either x ∈ V1 and y ∈ V2 OR y ∈ V1 and x ∈ V2.
- **Example of a bipartite graph.**
  - Group of Mobile phone subscribers
  - Group of Mobile phone providers
  - Choice of providers by subscribers
- **Example of a bipartite graph.**
  - Group of students V1
  - Group of courses V2
  - Choice of courses
- **A complete bipartite graph.** A bipartite simple graph in which every vertex of the first set (V1) is adjacent to every vertex of the second set (V2)

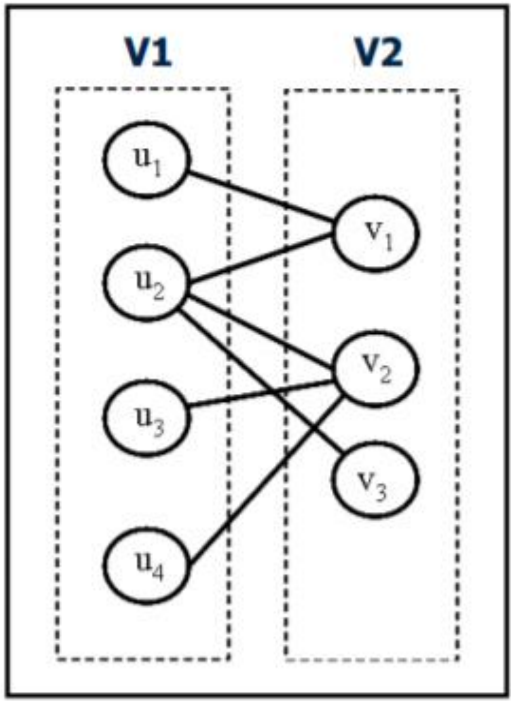


Fig 18. Representation of a Bipartite graph "G"

- Characteristics of the Bipartite graph G
  - V(G)=V1(G) U V2(G)
  - | V1(G)|=m, | V2(G)|=n
  - V1(G) ∩ V2(G) = Ø
  - No edges exist between any two vertices in the same subset vk(G), k=1,2

2.2. Basic concepts in graph theory

- 2.2.9 **Multipartite graph.** An Undirected graph G = (V, E) with following properties:
  - V can be partitioned into n sets V1 and V2, …, Vn.
  - The general notation is $K_{m1,m2,..,mn}$, where m1=|V1|, m2=|V2|,…, m_n=|Vn|,…, etc.
  - ***Exercise*.** Write each multipartite graph on this slide in the form $K_{m1,m2,..,mn}$ (e.g., $K_{1,1,1}$; $K_{3,3,3}$; $K_{3,3}$; $K_{2,2,2}$; $K_{2,2,2,2}$; $K_{1,2,2,5}$, etc.).

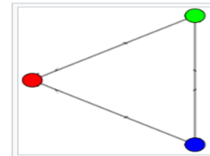
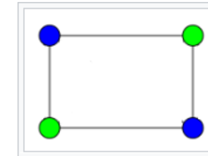
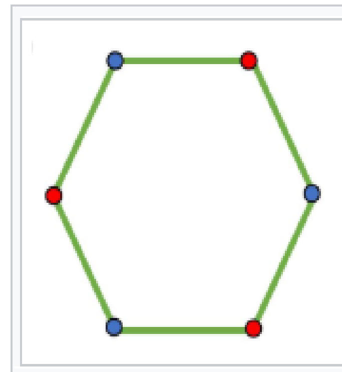
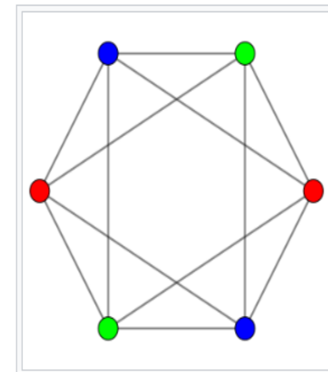
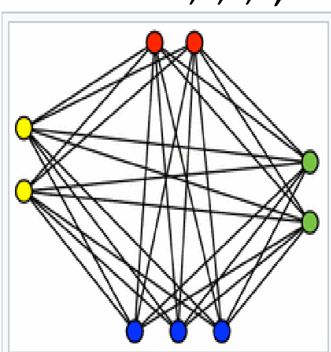
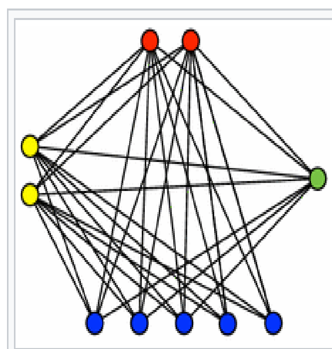
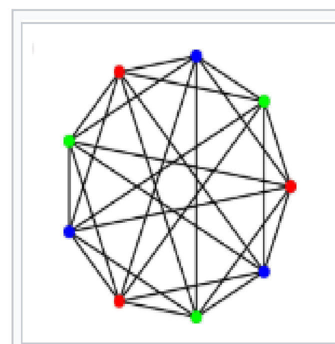
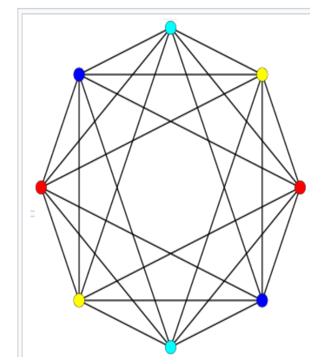









Fig 19. Schemes of multipartite graphs "G*i*"

2.2. Basic concepts in graph theory

- 2.2.10 Degree/Valency of a vertex/node;
  - ✓ Undirected graph: The number of edges incident on it.
  - ✓ Directed graph: The number of edges incident on it (In degree/valency); The number of edges leaving it (Out degree/valency)
  - ✓ **Exercise 1.** Give the degree of each vertex/node of the 4 graphs below.
  - ✓ **Exercise 2.** Determine the set of edges and the sets of nodes in each graph.
  - ✓ **Exercise 3.** Consider the notation G(V, E) in the case of directed graphs and explain the fundamental difference with G(V, E) in the case of undirected graphs.

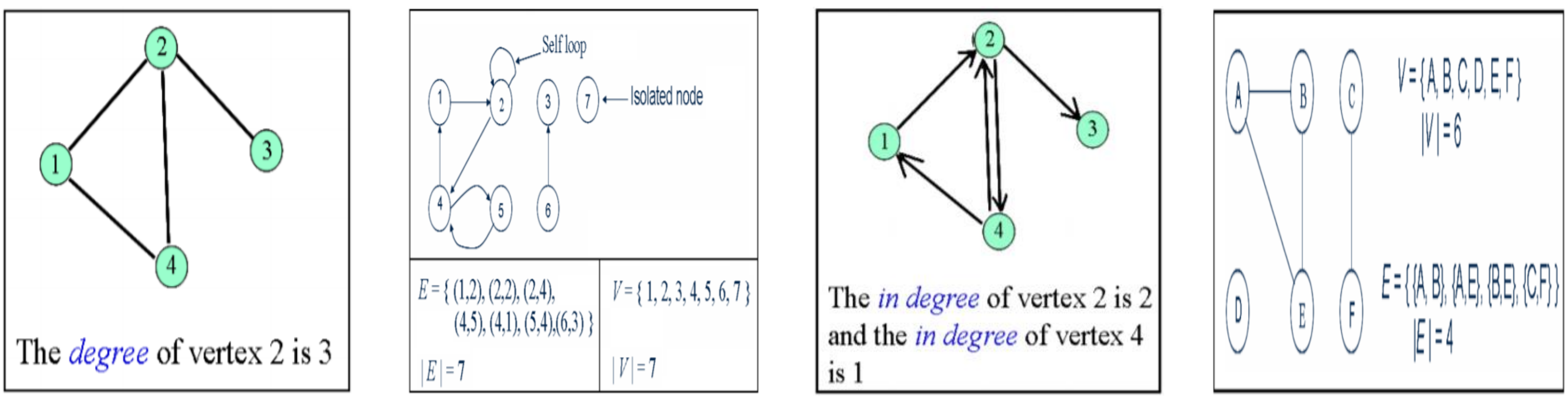
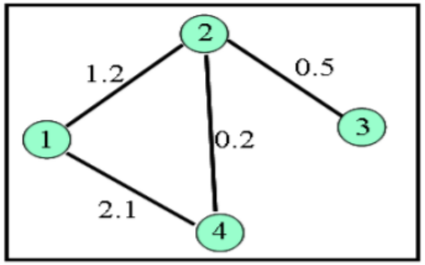


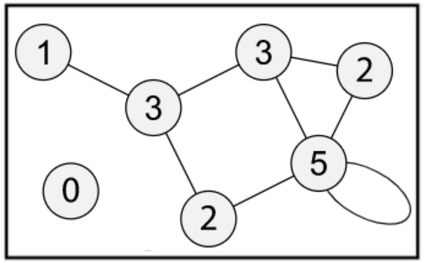
Fig 20. Schemes of undirected and directed graphs.

2.2. Basic concepts in graph theory

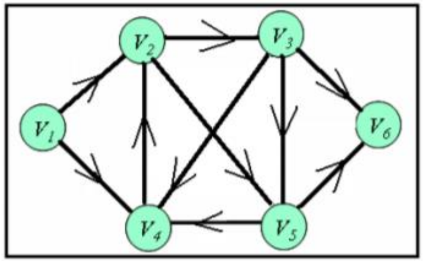
- 2.2.11 Valency sequence of a graph;
  - ✓ The set of valences of the vertices of the graph, usually arranged in non-decreasing order.
    - A vertex with valency “0” is an isolated vertex.
    - A vertex with valency “1” is an end-vertex.
    - A vertex with in-valency “0” is a source (see V1).
    - A vertex with out-valency “0” is a sink (see V6). .
    - A 0-valent graph is a graph with no connection (no edge). This graph is called a “null-graph” .
    - A “regular graph” is a graph with vertices having all the same valency.
  - ✓ **Exercise 1**. Determine the Degree/Valency of each vertex of the graphs in this slide.
  - ✓ **Exercise 2**. Determine the Valency-sequence of each graph.
  - ✓ **Exercise 3**. Determine the set of edges and the sets of nodes in each graph.



The valency-sequence is (1,2,2,3)



Give the valency-sequence



Give the valency-sequence

Fig 21. Schemes of undirected and directed graphs.

2.2. Basic concepts in graph theory

- 2.2.12 Tree; Subgraph; Spanning Subgraph; Spanning tree
  - ✓ A tree
    - A connected graph which contains no cycle
    - An acyclic, connected, undirected graph
  - ✓ A subgraph of a graph G=(V(G), E(G))
    - A graph H=(V(H), E(H)) such that V(H) is a part of V(G) and E(H) is a part of E(G).
  - ✓ A spanning subgraph
    - E(H) is a part of E(G) and V(H)=V(G). Under these conditions, H is a spanning subgraph of G.
  - ✓ A spanning tree
    - A spanning acyclic subgraph
    - A tree formed from all vertices of a graph
    - The number of distinct trees is n^{n-2}(Cayley tree formula); n is the order of the graph.
  - ✓ **<u>Exercise4</u>.** Is the subgraph in Fig. 22 a tree?. Give the key conditions for the subgraph in Fig. 22 to be: 1) A tree (i.e. non-spanning tree) and 2) A spanning tree.

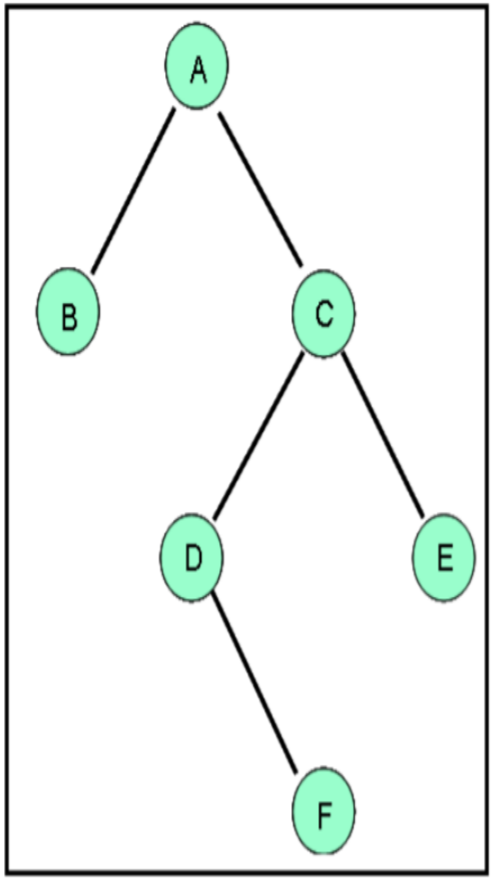


Fig 22a. Representation of a subgraph.

2.2. Basic concepts in graph theory

- 2.2.13 Trees of a directed graph

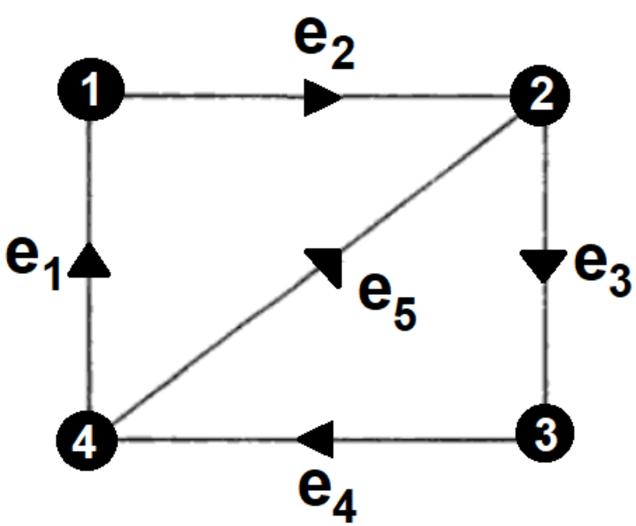


Fig 22b. Original graph: A directed graph

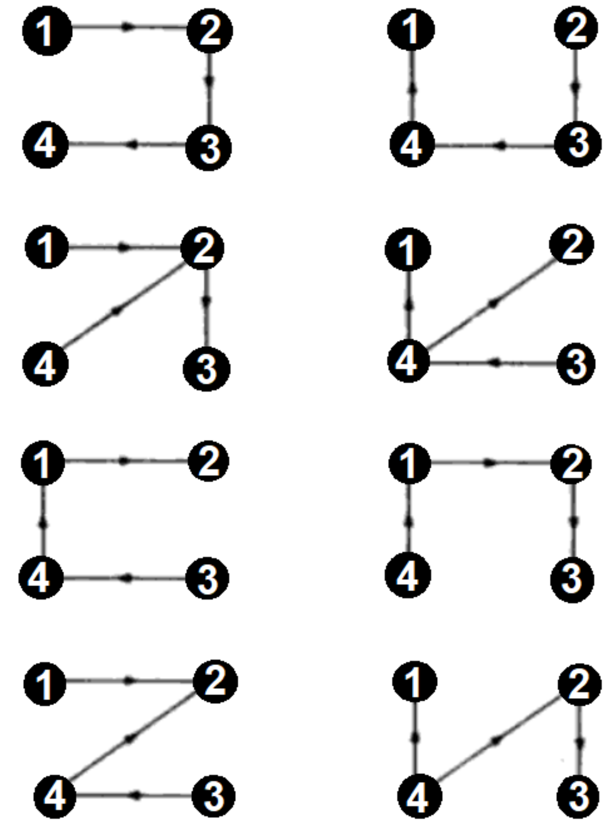


Fig 22c. Trees of the original graph

2.2. Basic concepts in graph theory

- 2.2.14 Rooted graph; Rooted tree
  - ✓ Rooted graph
    - A graph in which one vertex is distinguished from the rest
  - ✓ Rooted tree
    - A tree with one distinguished node (this node is called root-vertex or root)
  - ✓ **Exercise5.** Give the total number of spanning trees in graphs G1 and G2. Determine for each graph all possible spanning tress. Does the solution depend on the root?. Please justify your answer.

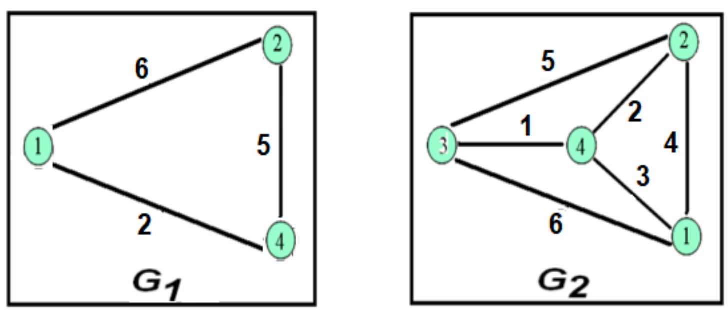


Fig 23a. Schemes of original graphs **G1** and **G2**

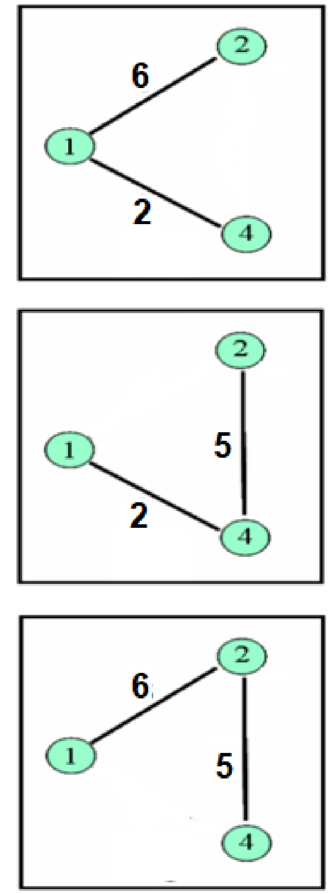
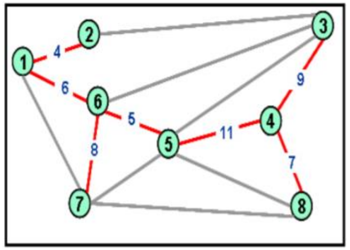
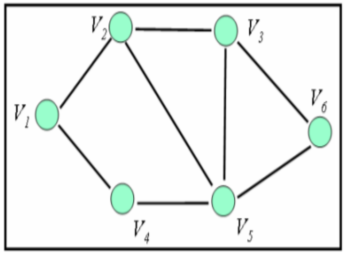


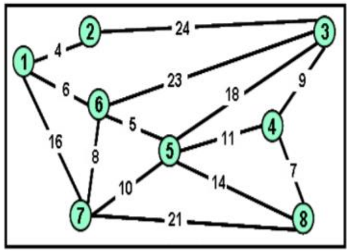
Fig 23b. Subgraphs of the graph **G1**

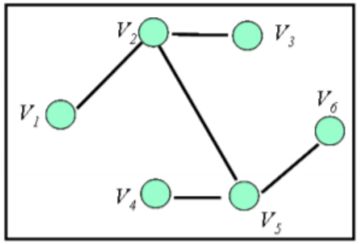
2.2. Basic concepts in graph theory

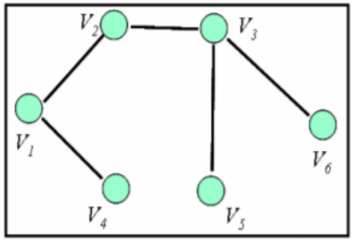
- 2.2.15 Shortest Path Spanning tree (SPST) and Minimum spanning tree (MST).
  - ✓ A SPST, T, is a spanning tree rooted at a particular node such that the |V |-1 minimum weight paths from that node to each of the other network nodes is contained in T.
  - ✓ MST is a spanning tree of a given graph whose sum of edge weights is minimized
  - ✓ **Exercise 6**. This slide contains 3 different graphs with their corresponding subgraphs corresponding to SPST and MST.
    - Identify each graph and the corresponding subgraphs.
    - Determine for each of the three graphs the SPST and the MST. Give the difference between SPST and MST.
    - What is the root that was considered to get/obtain each of the solutions in this slide?

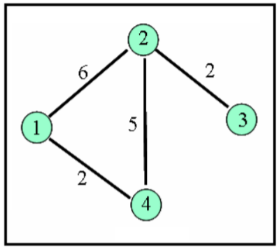


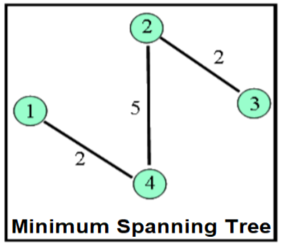












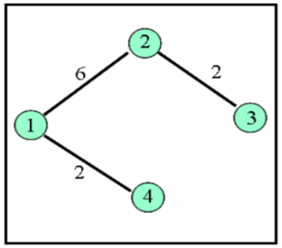


Fig 24. Schemes of 3 different graphs and corresponding subgraphs

2.2. Basic concepts in graph theory

- 2.2.16 Shortest Path Problems (SPP).
  - ✓ The SPP corresponds to finding a path from source (Node 1) to target/ destination (Node 2) in a graph such that the sum of the weights of edges constituting the path is minimized.
  - ✓ The shortest path problem (SPP) can be considered as an optimization problem over connected graph networks.
  - ✓ Example of SPP.
    - ○ Finding the shortest path between two junctions/intersections on a road map. Here the vertices/nodes correspond to intersections and the edges are the road segments. The weights correspond to traffic flow/ volume on each road segment.

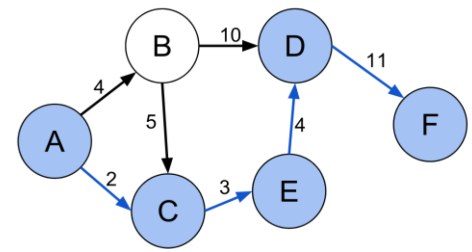


Fig 25a. Illustration of the SP in a directed graph

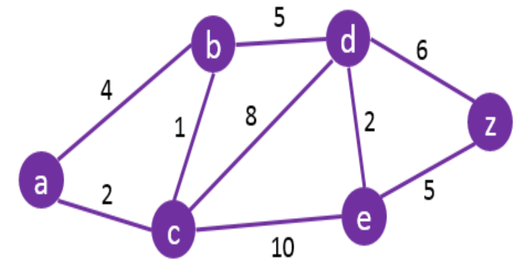


Fig 25b. An undirected weighted graph: Find the SP

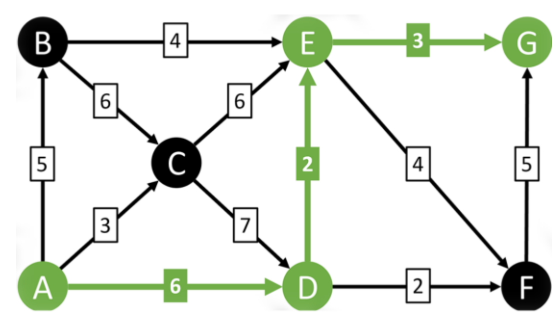


Fig 25c. Illustration of the SP in a directed graph

2.2. Basic concepts in graph theory

- 2.2.17 Traveling Salesman Problem (TSP).
  - ✓ TSP is defined as follows: "given a network of *n* cities where the salesman has to travel to each city exactly once and return to the starting city with minimum utilization of resources" (in general, this may be the shortest distance or in order words a "spanning" shortest path starting and ending at the starting city).
  - ✓ Sample applications of TSP in ITS.
    - Vehicle routing problems such as Pick-up and Delivery Problems (PDPs)
    - Vehicle and Crew Scheduling
    - A railway network and train schedules
    - Logistics and unmanned aircraft mission planning
    - Assignment and scheduling strategies for teams of unmanned vehicles.

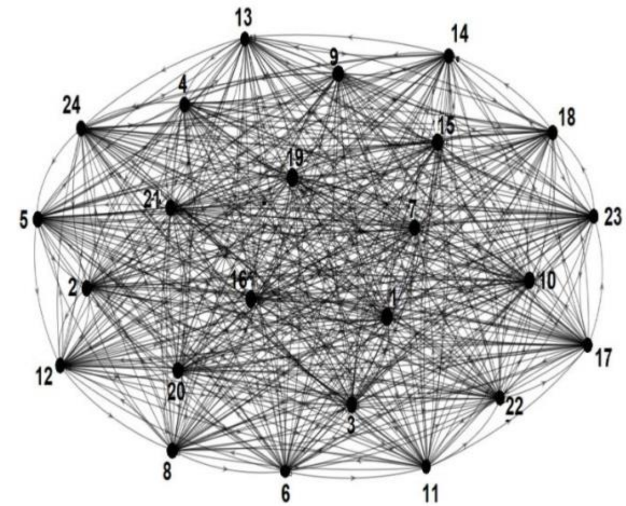


Fig 26a. Illustration of the TSP in an undirected graph

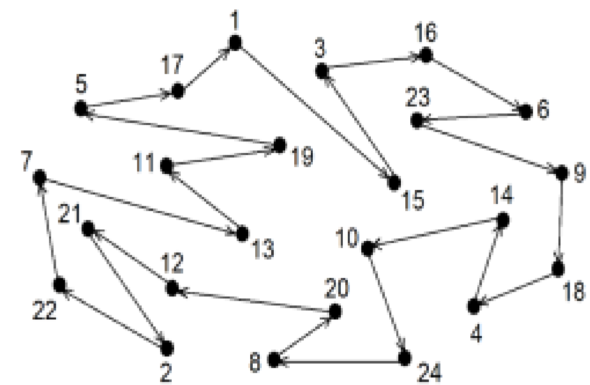


Fig 26b. Illustration of the TSP solution/tour in Fig. 26a

2.2. Basic concepts in graph theory

- 2.2.18 Flows in graphs
  - ✓ The flow problem corresponds to minimizing the total cost of sending multiple commodities across the graph network while satisfying all supplies and demands and respecting arc capacities.
  - ✓ <u>Key optimization constraints are related to:</u>
    - The Link capacity
    - The Flow conservation on transit nodes
    - The Flow conservation at the source
    - The Flow conservation at the destination
  - ✓ <u>Sample illustrations of multicommodity network flow (MCNF) problems.</u>
    - Message Routing in Telecommunication
    - Scheduling and Routing in Logistics and Transportation
    - Production Scheduling and Planning

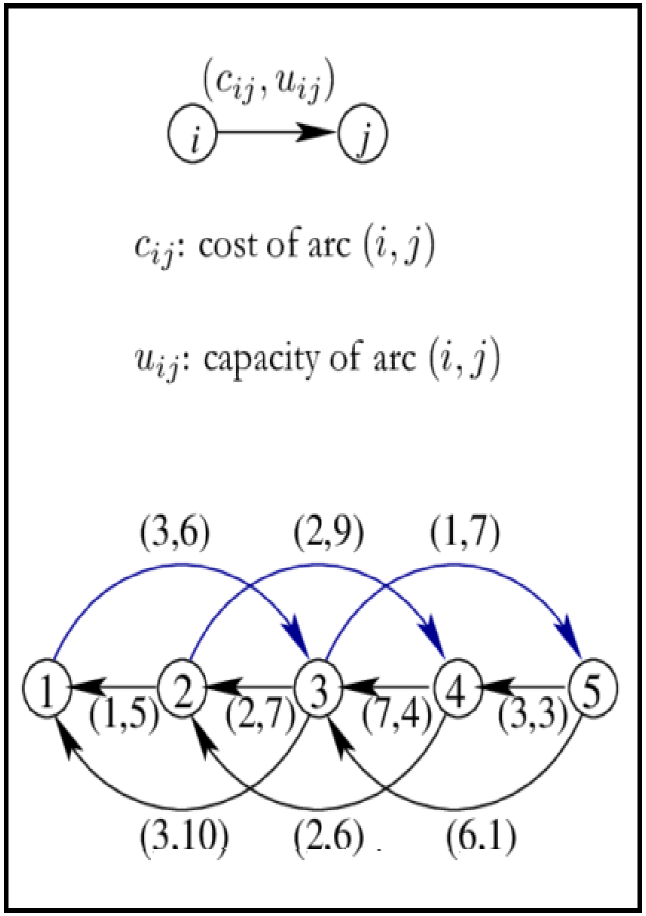


Fig 27. Illustration of a three-commodity Network Flows:

2.3. Dijkstra algorithm for the determination/detection of the SPST in graph networks concepts in graph theory

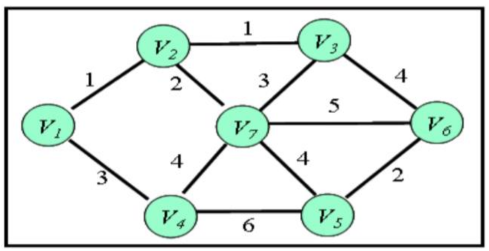
- 2.3.1 Dijkstra algorithm for SPST and MST
  - ✓ Assume non-negative edge weights
  - ✓ Given a weighted graph (G, W) and a node *s*, a shortest path tree rooted at *s* is a tree T such that, for any other node v ∈ G, the path between *s* and *v* in T is a shortest path between the nodes.
  - ✓ Examples of the algorithms that compute these shortest path trees are Dijkstra and Bellman-Ford algorithms as well as algorithms that find the shortest path between all pairs of nodes, e.g. Floyd-Marshall.

„Dijkstra Algorithm"

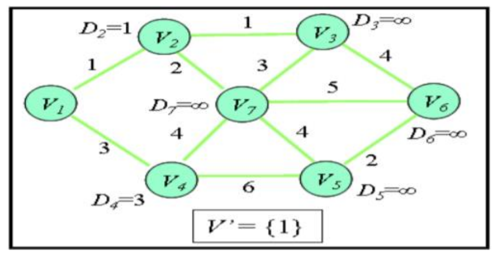
```
Procedure (assume s to be the root node)
V' = {s}; U = V - {s};
E' = ϕ ;
For v ∈ U do
    D_v = w(s, v);
    P_v = s;
EndFor
While U ≠ ϕ do
    Find v ∈ U such that D_v is minimal;
    V' = V' ∪ {v}; U = U – {v};
    E' = E' ∪ (P_v, v);
    For x ∈ U do
        If D_v + w(v, x) < D_x then
            D_x = D_v + w(v, x);
            P_x = v;
        EndIf
    EndFor
EndWhile
```

2.3. Dijkstra algorithm for the determination/detection of the SPST in graph networks concepts in graph theory

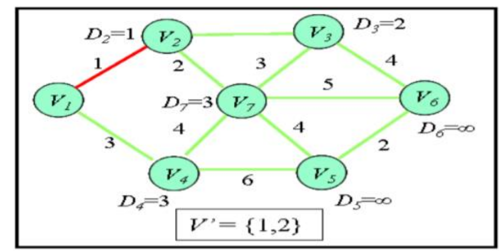
- 2.3.2 **Exercise.** Application of the Dijkstra algorithm for solving SPST and MST in a graph G.



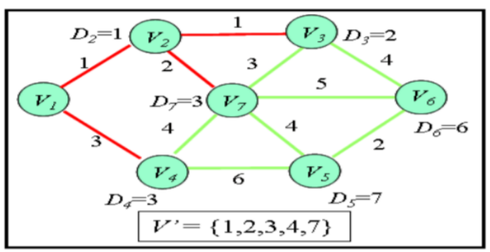
Original graph



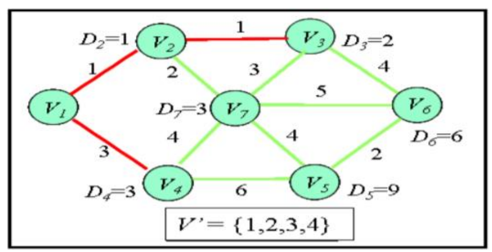
Step 1.



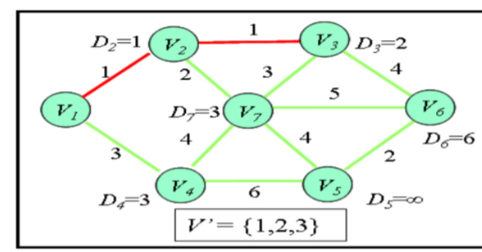
Step 2.



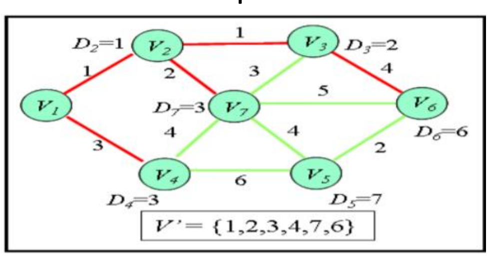
Step 5.



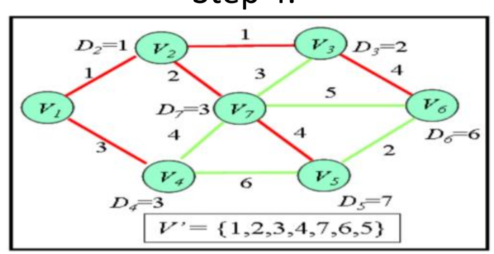
Step 4.



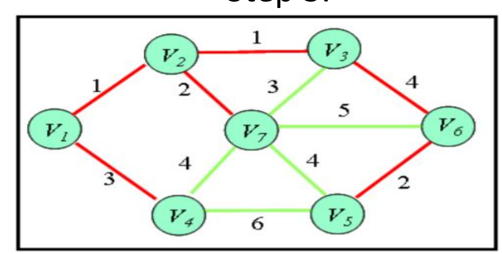
Step 3.



Step 6.



Step 7. **(SPST solution routed from *V1*)**



MST

Fig 28. An original graph with all possible SPST and MST solutions

2.3. Dijkstra algorithm for the determination/detection of the SPST in graph networks concepts in graph theory

- 2.3.3 Fundamental difference between SPST and MST: Some applications of MST
  - ✓ In a graph many SPST solutions are obtained/identified. The MST is not the "Shortest Path" but rather is a "Spanning Tree" with a total cost less than the total cost of the smallest SPST solution (identified amongst all possible SPST solutions).
  - ✓ Sample applications of MST
    - Designing physical networks (e.g., Road, telephone, electrical, hydraulic, TV cable, computer, etc.)
    - Cluster analysis (e.g., delete long edges leaves connected components)
    - Approximate solutions to NP-hard problems (e.g., Scheduling problems)
    - Unicast routing (one to one) → SPST
    - Multicast routing (one to several)
    - Maximum probability of reliable one to all communications → maximum weight spanning tree)
    - Load balancing → Degree constrained spanning tree
    - Indirect applications (e.g., learning salient features for real-time face verification, etc.).

2.4. Matrix- representation of graph networks

- The modeling of traffic processes is a very complex task
  - ✓ High degrees of nodes are involved/considered
  - ✓ Weights of links (edges) are very dynamic (i.e., change continuously)
  - ✓ The graphical representation is very dense.
  - ✓ Looking for optimal paths is a very difficult task due to the complex structure of graphs and uncertainties as well.
  - ✓ This justifies the need of computers to perform calculations
- Can computers understand graphs?
  - ✓ No computers can't understand graphs!!!!!!!; they only understand numbers. Therefore the matrix- representation of graphs appears of necessary importance.
- What is then the method to computationally cope with graphs?
  - ✓ The appropriate way/method to suit graph family into computer is to represent them into numbers that computer can understand.
  - ✓ A simple representation called Matrix (i.e. a table of numbers) can let the computer know about graph.

2.4. Matrix- representation of graph networks

- 2.4.1. Adjacency matrix: Case1. A Directed graph; Case 2. An Undirected graph
  - ✓ Adjacency Matrices. Suppose G is a graph with vertex $\{v_1, v_2, \ldots, v_n\}$. The adjacency matrix of G is the $n \times n$ matrix $A(G)=(a_{ij})$, where a_{ij} is the number of edges joining v_i and v_j.
  - ✓ The adjacency matrix is a matrix between vertex (in rows) and vertex (in columns)

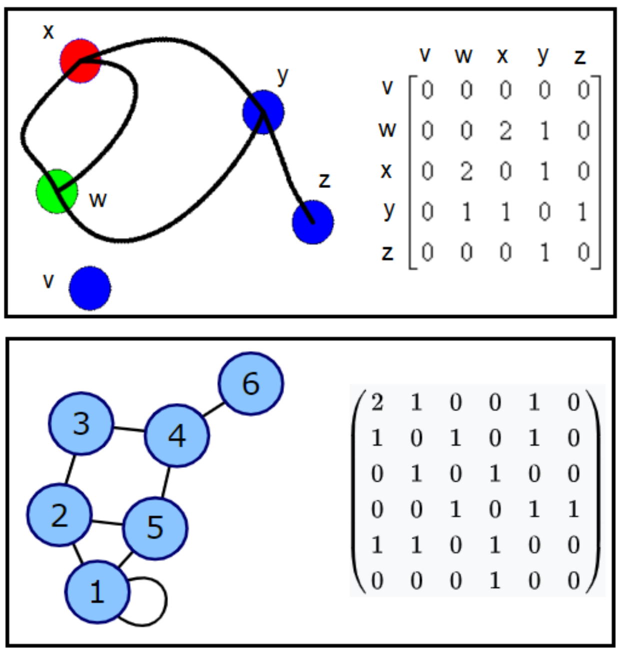


Fig 29. Undirected graphs with corresponding adjacency matrices

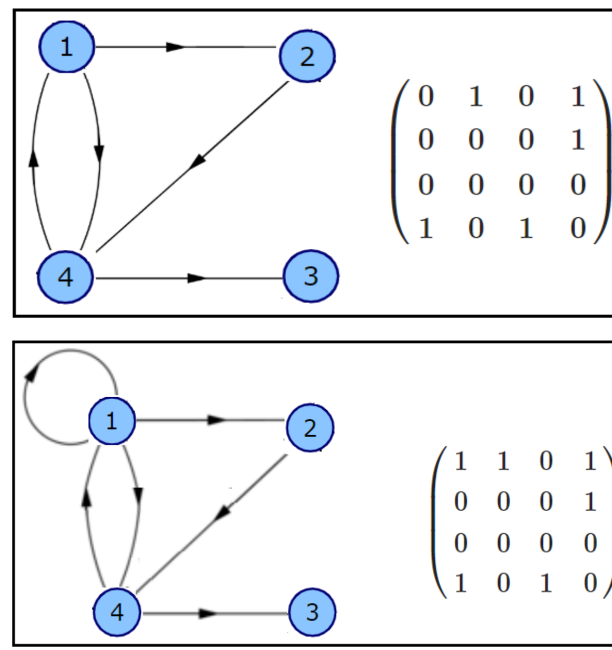


Fig 30. Directed graphs with corresponding adjacency matrices

2.4. Matrix- representation of graph networks

- 2.4.2. Incidence matrix: Case1. A Directed graph; Case 2. An Undirected graph
  - ✓ The incident matrix to represent a graph distinguishes edges
  - ✓ One of the ways to represent a graph into a matrix is by counting the connection between edge and vertices.
  - ✓ Instead of making matrix between vertex and vertex, the method makes matrix that related vertices to edges.
  - ✓ A vertex is said to be incident to an edge if the edge is connected to the vertex.
  - ✓ To fill the incident matrix, we look at the name of the vertex in row and name of the edge in column and find the corresponding of it. If a vertex is connected by an edge, we count number of leg in which the edge is connecting to this vertex and put this number as matrix element.

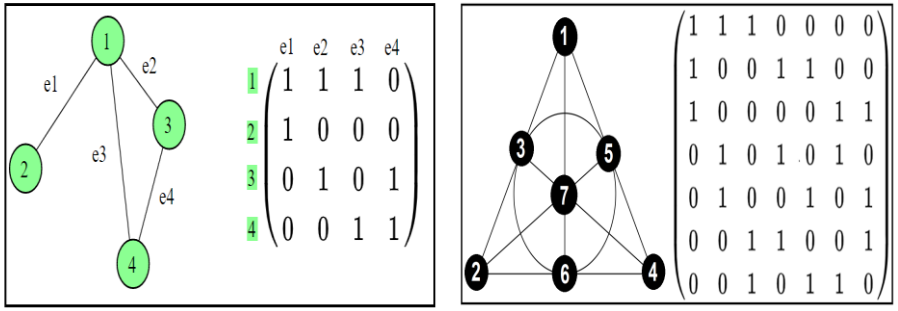


Fig 31. Undirected graphs with corresponding incidence matrices

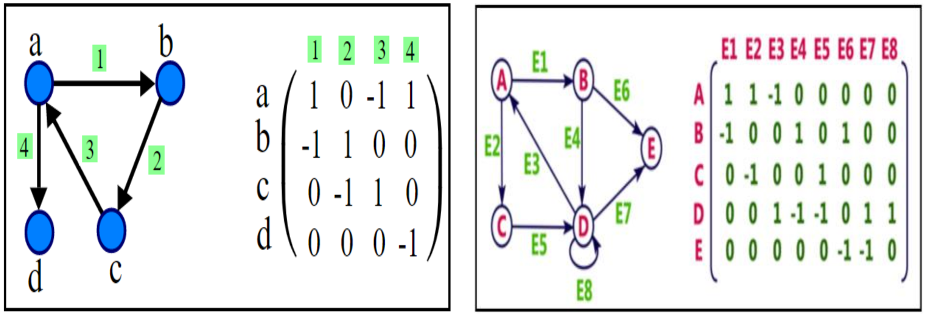


Fig 32. Directed graphs with corresponding incidence matrices

2.4. Matrix- representation of graph networks

- 2.4.3. Circuit matrix:
  - ✓ A graph without loop denoted by G = (V, E), which contains circuits.
  - ✓ The circuits of G are: C1, . . . , Cℓ . Thus, the circuit matrix of G is an ℓ × m matrix B = (bij) where
    - **Condition 1 (case of undirected graph):** b_{ij}=1 if the edge e_i is in the circuit C_i; b_{ij}=0 if the edge e_i is not in the circuit C_i.
    - **Condition 2 (case of directed graph):** b_{ij}=1 if the edge e_i is in the circuit C_i and in the same direction of rotation; b_{ij}=-1 if the edge e_i is in the circuit C_i and in the reverse direction of rotation; b_{ij}=0 if the edge e_i is not in the circuit C_i.

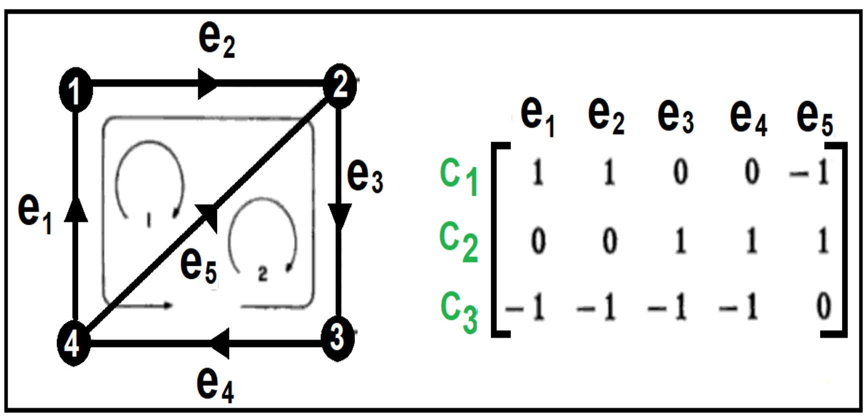


Fig 33. A directed graph with corresponding circuit matrix

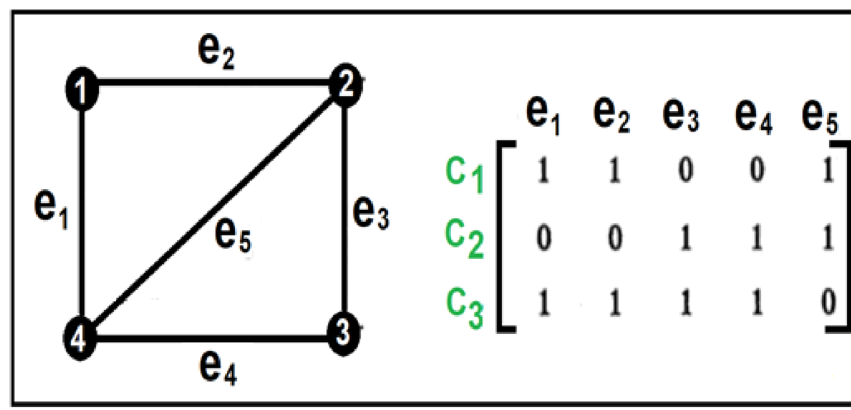


Fig 34. An undirected graph with corresponding circuit matrix

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Chapter 3.

Mathematical modeling of traffic flow

(Chapter’s detailed description)

Intelligent Transport Systems: New ICT – based Master’s Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

3.1. Fundamental parameters of traffic flow and related challenges

- ✓ The focus here is on the description/analysis of the relationships between the fundamental parameters of traffic (i.e., flow, speed and density). Specifically the correlation between the parameters is analyze in all states of traffic namely the state under saturation, the state at saturation and the state over saturation. Finally, we discuss how difficult it is to depict the oversaturated state of traffic based on the fundamental parameters of traffic.

3.2. Mathematical modeling of the fundamental parameters of traffic flow

- ✓ Different mathematical models expressing the relationship between the fundamental parameters of traffic (flow, speed, and density) are presented and are used to depict and predict different states of traffic namely undersaturated, saturated and oversaturated traffic.
  - Single regime models are considered such as Greenshields, Drake, Daganzo, Smulders, Greenberg, Drew, Pipes and Munjal. These models are further analyzed.
  - Multi regime models are further considered such as the Edie model and the modified Greenberg model. The analysis of these models is carried out.

3.3. Mathematical modeling of the car-following theory of traffic flow and analysis of the dynamics of headways (both space- and time- headways)

- ✓ The car-following theory corresponds to the microscopic traffic analysis. The mathematical model of traffic is expressed in the form of coupled ordinary differential equations (ODE) of order 2. Each ODE of order 2 corresponds to the equation of motion of a specific vehicle. This equation expresses the acceleration of the vehicle and this acceleration is function of three main factors namely, the space headway, the position of vehicles, and the speeds of vehicles. This latter factor (i.e. speed) is correlated/depends to/on the driver behavior.
- ✓ The analysis of the headways between vehicles leads to the depiction/prediction of traffic states (e.g., under saturated, saturated and oversaturated states).

3.4. Mathematical modeling of traffic flow on a single lane road segment (No overtaking)

- ✓ This analysis corresponds to macroscopic traffic analysis. Different PDE models such as the Lighthill-Whitham-Richard (LWR), Ross, Payne, and Luis are considered to analyze the dynamics of traffic flow on a road segment made up of a single road. This corresponds to a traffic dynamics without overtaking.
- ✓ The performances of the models at stake are further compared, and a good agreement is obtained in the state of traffic under saturation. In contrast a divergence is observed between the models while performing oversaturation.

3.5. Mathematical modeling of traffic flow on a double lane road segment (with overtaking and without ramps)

- ✓ This analysis corresponds to macroscopic traffic analysis. The extended LWR model is used for the modeling of traffic flow on multi-lanes road segments with overtaking. The extended LWR is expressed into the form of coupled PDEs models, which are further used to express the coupling between vehicles moving into different lanes on a multi-lane road segment. The coupling between vehicles in different lanes is characterized/expressed by lane change and overtaking.

3.6. Mathematical modeling of traffic flow on a double lane road segment (with both overtaking and ramps)

- ✓ This analysis corresponds to macroscopic traffic analysis. The basic METANET model expressed into two coupled PDE models is used for the modeling of traffic flow on a two-lane road segments with both overtaking and ramps. The effects of traffic flow in both exit- and Entrance- ramps on the dynamics of traffic flow in the main road (made-up of two lanes) is further analyzed.

3.7. Generalization: Mathematical modeling of traffic flow on a multilane road segment

- ✓ This analysis corresponds to macroscopic traffic analysis. As a generalization, the basic METANET model expressed into coupled PDE models is used for the modeling of traffic flow on multilane road segments with both overtaking and ramps. The effects of traffic flow in both exit- and Entrance- ramps on the dynamics of traffic flow on each lane of the main road (made-up of multiple lanes) is further analyzed.
- ✓ The dynamics of traffic flow on each lane of the multilane road segment is modeled mathematically by a first order partial differential equation (PDE).
- ✓ Finally, the performance of the METANET PDE model is compared with the performances of the classical Greenshields model. Both convergence and divergence are observed between the performances. In the traffic state under saturation as well as the traffic at saturation, a very goo agreement is observed between the performances. In contrast when the traffic state exceeds the state of saturation (say, traffic oversaturation) a divergence is observed between the performances of the PDEs-model and the performances of the classical Greenshields model.

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Chapter 4.
Basics of traffic signals control at isolated junction

(Chapter’s detailed description)

Intelligent Transport Systems: New ICT – based Master’s Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

4.1. Performance criteria of a traffic junction

- ✓ The performance criteria of traffic junctions are defined and are described (e.g. Green signal splitting, Cycle time/length, Throughput of junctions, Delay at junctions, Number of stops at junctions, quality of service of junctions, etc.).

4.2. Mathematical model of a traffic junction

- ✓ The mathematical model of traffic junction is expressed by a set of coupled ordinary differential equations. The dependent variables are the green signals of different phase groups of the traffic junction.

4.3. Identification of a traffic junction

- ✓ A traffic junction is identified though: * the architecture/geometry, * the number of streams, * the number of lanes per stream, * the number of phase groups, * the control strategy (pretimed, semi-actuated, actuated, and roundabout), etc.

4.4. Classification of traffic into streams

- ✓ A road segment is split up into streams and a stream is defined through a specific lane grouping strategy. The aim here is to consider a complex traffic junction and demonstrate how the streams can be defined/designed depending on the specific control strategy chosen for the traffic junction. Finally we demonstrate that a road segment can be classified into different streams (one, two, three, ..., multiple, etc.).

4.5. Phase- groups

- ✓ The aim is to demonstrate how traffic signals at junction can be classified into phase groups. This corresponds to the phase-grouping procedure. It is worth noting that the phase-grouping process significantly affects the performance of a junction.

4.6. Traffic signal phasing and timing plan

- ✓ The aim is to discuss the criteria used to classify traffic signals into phase groups and discuss the conditions to assign different durations of green signals to specific phase groups. It should be explained why the phase groups are assigned different green signals time-durations (called signals timing or timing plan strategy).

4.7. Protected- and Unprotected- turns

- ✓ The aim here is to explain why a green signal serving a left turn traffic can be protected. Conditions for the protection of the left turn traffic are given/formulated.

4.8. Critical lane concept

- ✓ The aim here is to describe/define the conditions/criteria which are used to qualify a lane as critical. Amongst these conditions one can enumerate the volume of traffic carried by the lane, the speed of traffic through the lane, etc.

4.9. Cycle length; Green time; All-red interval; Delays; Dilemma zones; Pedestrian crossing time; Level of service (LOS); Some illustrative examples from practice.

- ✓ The above keywords are defined/described, and concrete examples are provided for illustration. Finally concrete traffic junctions of different geometries are envisaged, and the above keywords are expressed/evaluated for each traffic junction.

4.10. Graphs for traffic lanes and lane- groups

- ✓ The traffic at junction is modelled into graphs. Here the lanes are nodes and the weights are the traffic volume in lanes. The advantage of modeling a traffic in the form of graphs in also clearly demonstrated.

4.11. Graphs for road intersections

- ✓ A traffic network made up of several junctions is considered. The conditions for junctions to be considered as coupled junctions are discussed. Finally traffic network is model in the form of graph. The nodes are represented by junctions and the weights are represented by the traffic volume carried by a road segment connecting two junction. The graph in this case is a bi-directional graph.

Chapter 5.

Mathematical modelling of scenarios/events in Railway transportation

(Chapter’s detailed description)

5.1. Graphical models of specific examples in Railway transportation

- ✓ Many scenarios/problems related to railway transportation are considered and are modeled in the form of graphs.
  - Train scheduling,
  - Railway Rescheduling Problem,
  - Multimodal freight terminal networks,
  - Urban train networks,
  - Optimization problem and reduction of the cost of distribution and transportation of goods.
  - Multicommodity Network Design Problem in Rail Freight Transportation Planning,
  - Selection of transport facilities for multimodal freight transportation, optimal empty rail car distribution at railway transport nodes,
  - Planning the distribution of locomotives to meet the demand for making up trains

5.2. Mathematical models of specific examples in Railway transportation

- ✓ All the scenarios/problems considered in the section above (see 4.9.) are also modelled mathematically and the mathematical models are obtained.

Co-funded by the
Erasmus+ Programme
of the European Union

Chapter 6.
Basics of supply chain networks (SCN) and modelling principles

(Chapter's detailed description)

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

6.1. Supply chain management (SCM): Integration and management of business processes

- ✓ This section provides the description of the functioning principle of supply chain networks (SCNs). An example of SCN encompassing the integration and management of business processes is considered and is fully described.

6.2. Structure of a SCN

- ✓ Presentation of the general structure of a supply chain network (SCN).

6.3. Framework for SCM

- ✓ Description of the framework of supply chain management (SCM).

6.4. Different types of intercompany business process links

- ✓ Enumeration of the different types of intercompany business process links.

6.5. Different types of intercompany business process links brackdown

- ✓ Background of different types of intercompany business process links.

6.6. Fundamental management components in a supply chain network

- ✓ Description of components of a supply chain management (SCM).

6.7. General design principle of a SCN

- ✓ Explanation of the design principle of a supply chain network (SCN).

6.8. Graphical modeling of a SCN

- ✓ Explanation of the general principle of modelling a supply chain network in the form of graph. Two concrete application examples of supply chain networks are considered to illustration the concept.

6.9. Mathematical modelling of a SCN

- ✓ Explanation of the general principle of modelling a supply chain network mathematically. Two concrete application examples of supply chain networks are considered to illustration the concept.

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

Chapter 7.
MATLAB-CODING: Numerical simulation of Microscopic, Macroscopic and Mesoscopic traffic dynamics
(Chapter's detailed description)

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

7.1. Case 1. Microscopic traffic dynamics modelled by ordinary differential equations

- ✓ Consideration of a traffic scenario at a microscopic level of detail.
- ✓ Explanation of the modelling principle of the scenario at stake in the form of ordinary differential equations (ODE)
- ✓ Explanation of the use of the numerical simulation tool MATLAB for the computation/solving of the ordinary differential equation (ODE) obtained.
- ✓ The solution of the ODE expresses the fonctioning principle of the traffic scenario.

7.2. Case 2. Macroscopic traffic dynamics modelled by partial differential equations (PDEs)

- ✓ Consideration of a traffic scenario at a macroscopic level of detail.
- ✓ Explanation of the modelling principle of the scenario at stake in the form of partial differential equations (PDE)
- ✓ Explanation of the use of the numerical simulation tool MATLAB for the computation/solving of the partial differential equation (PDE) obtained.
- ✓ The solution of the PDE expresses the fonctioning principle of the traffic scenario.

7.3. Case 3. Mesoscopic traffic dynamics modelled by the coupling between ODEs and PDEs

- ✓ Consideration of a traffic scenario at a mesoscopic level of detail. This scenario is modelled mathematically by a coupled system encompassing both ODE and PDE.
- ✓ Use of MATALB to solve the coupled system; use the solution to analyse the scenario.

Chapter 8.
LAB: SYNCHRO 7 & 9: Design of traffic junctions with different control strategies using SYNCHRO and measurement of the performance criteria of various traffic junctions
(Chapter’s detailed description)

Intelligent Transport Systems: New ICT – based Master’s Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP

8.1. Presentation of the traffic signals simulation software SYNCHRO 7 & 9

- ✓ Description of the functioning principle
- ✓ Description of the design principle of an isolated traffic junction
- ✓ Explanation of the viewing and measurement of the performance criteria of an isolated traffic junction (e.g. Green signal splitting, Cycle time/length, Throughput of junctions, Delay at junctions, Number of stops at junctions, quality of service of junctions, etc.).

8.2. Case 1. Pretimed control

- ✓ Use SYNCHRO 7 & 9 for the design of a T-junction and measurement of performance criteria of the isolated traffic junction under the pretimed control strategy.
- ✓ Use SYNCHRO 7 & 9 for the design of a 4-segment isolated junction and measurement of performance criteria of the traffic junction under the pretimed control strategy.

8.3. Case 2. Actuated control

- ✓ Same analysis as in 8.2 Case 1.

8.4. Case 3. Semi-actuated control

- ✓ Same analysis as in 8.3 Case 2.

8.5. Case 4. Roundabout

- ✓ Same analysis as in 8.4 Case 3.

INTRAS

Co-funded by the Erasmus+ Programme of the European Union

END

OF

LECTURE 1

Intelligent Transport Systems: New ICT – based Master's Curricula in Uzbekistan (INTRAS)
Agreement number: 2017-3516/001-001
Project reference number: 586292-EPP-1-2017-1-PL-EPPKA2-CBHE-JP