**“Mathematical methods in transportation”**

**Syllabus CT.03/1**

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Version 1

# Name of the course

**Mathematical methods in transportation**

# ECTS credits

6 Credits, (**45 hours of Theory + 30 hours of Exercises & Lab**), 1st semester

# Objectives

This lecture gives students basic technical knowledge a series of mathematical methods of relevance for traffic, transportation, and logistics. A particular attention is devoted to the modeling processes in transportation and logistics. Models are obtained/derived to explain and control/optimize the dynamics of systems, scenarios, and phenomena in transportation.

Theoretical concepts/methods are developed to model/represent given systems, scenarios, and phenomena in transportation. The developed models are expressed into graphical and mathematical forms.

Overall, the main objectives of this lecture are expressed by the following keywords: *Methods and models in transportation; Traffic and transport; Supply chains and logistics; Numerical simulation of mathematical and graphical models.*

The Lecture allows full understanding of the key graph theoretical concepts. Further the application of graph theory for the modelling of systems, scenarios and phenomena in transportation is considered through some real-world case studies selected in the following fields: Road transportation- Railway transportation – Communication engineering – Supply chain and Logistics.

The mathematical modeling of systems, scenarios and phenomena in transportation is further considered. Several case studies related to Railway Transportation, Road Transportation, Supply Chain and Logistics are considered. Full detail of the mathematical modeling procedure is provided and the corresponding mathematical modeled are obtained.

Finally the aforementioned mathematical models are used to study the dynamical behavior of the corresponding systems, scenarios and phenomena. The main advantage in this context is to demonstrate how mathematics can be used to analyze, understand, control and predict (or forecast) the deterministic and/or stochastic behavior of real-world systems, scenarios and phenomena in transportation.

# Learning outcomes

The general expectation regarding the knowledge to be provided/acquired is as follows:

* Understanding of basic systems, scenarios and phenomena in transportation.
* Mastering of the basics concepts of systems’ modeling in transportation
* Mastering of the modeling of transportation systems scenarios and phenomena into graphical forms.
* Mastering of the modeling of transportation systems, scenarios and phenomena into mathematical forms.
* Mastering of the modeling of shortest path problems with applications in in both road- and railway-transportation.
* Mastering of the modeling of traveling salesman problems with applications in both road- and railway- transportation.
* Mastering of the mathematical modeling of traffic flow at macroscopic level using Partial differential equations (PDEs); Applications in practice for the modeling of real traffic scenarios on arterial roads.
* Mastering of the mathematical modeling of traffic flow at microscopic level using Ordinary Differential Equations (ODEs); Applications in practice for the modeling of real traffic scenarios on arterial roads.
* Acquiring some basic knowledge in computational engineering. Specifically, the use of MATLAB/SIMULINK for scientific computing (e.g. numerical simulation of nonlinear Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs))
* Understanding of the functioning principle of supply chain networks (SCN) and their modeling principle.
* Acquiring some basic knowledge in logistics and scheduling.

# Contents

1. General introduction
	1. Mathematics as important instrument for improving traffic and transport
	2. Definition of some keywords and concepts in transportation
	3. Principles of modeling nonlinear dynamical systems in transportation
	4. Examples of systems' models in transportation
	5. Mathematical modeling in transportation: Pros, Cons and related challenges
	6. Fundamentals of system theory
	7. Evaluation of modeling techniques in Transportation
	8. Sample illustration of real-life nonlinear dynamic systems/scenarios in transportation
	9. Transportation systems/scenarios undergoing nonlinear and time varying dynamics
2. Basics of graph theory and applications in transportation
	1. Sample applications of graph theory in transportation.
		1. Applications in communication engineering
		2. Applications in road transportation
		3. Applications in railway transportation
		4. Applications in supply chain networks and logistics
	2. Basic concepts in graph theory
		1. Magnitude; Size; Edges (links) Weights (costs of Edges); total cost of a graph network
		2. Directed/Undirected graph; Weighted/Unweighted graph; Completed/Uncompleted graph; Connected/Disconnected graph; Bidirectional graphs; Real graphs; Virtual graphs;
		3. Walk; Closed walk; Trail; Path; Cycle; Loop; Forest; Cyclic graph; Acyclic graph; Bipartite graph; Multipartite graph; Loop (or Self-loop); Length of a walk
		4. Valency of a vertex; Valency sequence; Degree of a vertex/vertice; Adjacent vertices; Isolated node/vertex; Subgraph; Tree; Spanning tree; Rooted graph; Rooted tree
		5. Minimum spanning tree (MST); Shortest path (SP); Shortest path spanning tree (SPST); Traveling salesman problem (TSP); Symmetric TSP; Asymmetric TSP
		6. Flows in graphs + one algorithm for “Max Flow” determination in a graph
	3. Dijkstra algorithm for the determination/detection of the SPST in graph networks
		1. SPST
		2. MST
		3. Fundamental difference between SPST and MST
	4. Matrix- representation of graph networks
		1. Adjacency matrix: Case1. A Directed graph; Case 2. An Undirected graph
		2. Incidence matrix: Case1. A Directed graph; Case 2. An Undirected graph
		3. Circuit matrix: Case1. A Directed graph; Case 2. An Undirected graph
3. Mathematical modeling of traffic flow
	1. Fundamental parameters of traffic flow and related challenges
	2. Mathematical modeling of the fundamental parameters of traffic flow
		1. Single regime models (Greenshields, Greenberg, Drew, Pipes and Munjal)
		2. Multi regime models (Edie and Modified Greenberg)
	3. Mathematical modeling of the [car-following theory of traffic flow](http://cedb.asce.org/CEDBsearch/record.jsp?dockey=0013088) and analysis of the dynamics of headways (both space- and time- headways)
	4. Mathematical modeling of traffic flow on a single lane road segment (No overtaking)
	5. Mathematical modeling of traffic flow on a double lane road segment (with overtaking and without ramps)
	6. Mathematical modeling of traffic flow on a double lane road segment (with both overtaking and ramps)
	7. Generalization: Mathematical modeling of traffic flow on a multilane road segment
4. Mathematical modelling basics of traffic signals control at an isolated junction
	1. Performance criteria of a traffic junction
	2. Mathematical model of a traffic junction
	3. Identification of a traffic junction
	4. Classification of traffic into streams
	5. Phase - groups
	6. Traffic signal phasing and timing plan
	7. Protected- and Unprotected- turns
	8. Critical lane concept;
	9. Cycle length; Green time; All-red interval; Delays; Dilemma zones; Pedestrian crossing time; Level of service (LOS); Some illustrative examples from practice.
	10. Graphs for traffic lanes and lane- groups
	11. Graphs for road intersections
5. Mathematical modelling of scenarios/events in Railway transportation
	1. Graphical models of specific examples in Railway transportation
	2. Mathematical models of specific examples in Railway transportation
6. Basics of supply chain networks (SCN) and modelling principles
	1. Supply chain management (SCM): Integration and management of business processes
	2. Structure of a SCN
	3. Framework for SCM
	4. Different types of intercompany business process links
	5. Different types of intercompany business process links background
	6. Fundamental management components in a supply chain network
	7. General design principle of a SCN
	8. Graphical modeling of a SCN
	9. Mathematical modelling of a SCN.
7. MATLAB-CODING: Numerical simulation of Microscopic, Macroscopic and Mesoscopic traffic dynamics
	1. Case 1. Microscopic traffic dynamics modelled by ordinary differential equations (ODEs)
	2. Case 2. Macroscopic traffic dynamics modelled by partial differential equations (PDEs)
	3. Case 3. Mesoscopic traffic dynamics modelled by the coupling between ODEs and PDEs
8. LAB: SYNCHRO 7 & 9: Design of traffic junctions with different control strategies using SYNCHRO and measurement of the performance criteria (e.g. Green signal splitting, Cycle time/length, Throughput of junctions, Delay at junctions, Number of stops at junctions, quality of service of junctions, etc. ) of various traffic junctions
	1. Case 1. Pretimed control
	2. Case 2. Actuated control
	3. Case 3. Semi-actuated control
	4. Case 4. Roundabout

# Teaching method

Lectures, case studies, Tutorials/exercises, Numerical coding using MATLAB, DEMO. USING SYNCHRO 7 & 9.

* The slides are available for the whole lecture. These slides must be provided to students (or must be uploaded in the MOODLE system). The full contents of each slide is systematically explained by the Lecturer. Additional examples which are not included in slides will be proposed by the Lecturer to allow good understanding of the information provided.
* The slides contain exercises with solutions for the good understanding of the content of each chapter. These solutions are systematically explained (during the lecture) by the Lecturer.
* The Slides contain exercises without solutions to be solved by students during the lecture (this is part of oral exam). The students are fully assisted by the Lecturer in order to obtain correct/exact solutions to the proposed exercises. This will help to check whether the students have understood the chapters or not.
* Several exercises will be proposed by the Lecturer to be solved by students as projects. This will help to test the self-learning potential of students.

# Assessment method

Mid-term and final oral and/or written examination, exercises from case studies.

# Textbooks - Publications - Software

**Textbooks**

* Treiber M., and Kesting A. (2013). **Traffic Flow Dynamics: Data, Models and Simulation**, Springer-Verlag, Berlin Heidelberg, ISBN 978-3-642-32460-4,
* Haight F. A., (2012). [**Mathematical Theories of Traffic Flow**, Academic Press,](https://www.amazon.com/Mathematical-Theories-Traffic-Frank-Haight/dp/0124110053/ref%3Dsr_1_5?s=books&ie=UTF8&qid=1538662483&sr=1-5) ISBN-13: 978-0124110052
* Cascetta E. (2009). **Transportation Systems Analysis: Models and Applications**, Second Edition, Springer.
* Bondy J. A. and Murty U. S. R. (1982). **Graph theory with applications**, Fifth Edition, Elsevier Science Publishing Co.
* [Watson](http://www.worldcat.org/search?q=au%3AWatson%2C+Michael%2C&qt=hot_author) M.; [Lewis](http://www.worldcat.org/search?q=au%3ALewis%2C+Sara%2C&qt=hot_author) S.; [Cacioppi P.](http://www.worldcat.org/search?q=au%3ACacioppi+Peter%2C&qt=hot_author" \o "Search for more by this author) ; [Jayaraman](http://www.worldcat.org/search?q=au%3AJayaraman%2C+Jay%2C&qt=hot_author) J. (2014). **Supply chain network design : applying optimization and analytics to the global supply chain**, Second Edition, New Jersey Pearson Education.
* D. Davendra (2010). **Traveling Salesman Problem, Theory and Applications**, InTech, ISBN 978-953-307-426-9.

**Selected relevant Publications**

* J. C. Platt and A. H. Barr, Constrained differential optimization for neural networks, Dept. Comput. Sci., California Inst. Technol., Pasadena, CA, USA, Tech. Rep. TR-88-17 (1988).
* F. Araujo, B. Ribeiro and L. Rodrigues, A neural network for shortest path computation, IEEE transactions on neural networks, vol. 12, No. 5 (2001) 1067-1073.
* N. Alireza and O. Farahnaz, An efficient dynamic model for solving the shortest path problem," Transportation research part c, vol. 26 (2013) 1-19.
* J. C. Chedjou and K. Kyamakya, Benchmarking a Recurrent Neural Network Based Efficient Shortest Path Problem (SPP) Solver Concept under Difficult Dynamic Parameter Settings Conditions, NeuroComputing, vol. 196 (2016) 175-209.
* J. C. Chedjou and K. Kyamakya, A Universal Concept Based on Cellular Neural Networks for Ultrafast and Flexible Solving of Differential Equations, IEEE Transactions on Neural Networks and Learning Systems, vol. 26, No. 4 (2015) 749-762.
* M. Larrañaga, J. Anselmi, U. Ayesta, P. Jacko, and A.Rom, Optimization Techniques Applied to Railway Systems( 2013) <hal-00780524>.
* C. Strotmann, Railway scheduling problems and their decomposition, Thesis/Dissertation (2007), University of Osnabrück, Germany.
* S. Göttlich, M. Herty, and A. Klars, Network Models for Supply Chains International Press, vol. 3, No. 4, (2005) 545–559.
* D. Huisman, L. G. Kroon, R. M. Lentink, and M. J.C.M. Vromans, Operations Research in Passenger Railway Transportation, Econometric Institute Netherlands (2005), Report EI2005-16

Software

**\*** Numerical Computing: Algorithmic/Coding and Model-Based Design

* **MATLAB TAH FULL SUITE** for scientific computing, MathWorks (1984), USA

https://de.mathworks.com/academia/student\_version.html

Traffic Simulation tools

**\*** Geometric design

* **Synchro 10 plus SimTraffic**: Signal Timing and Analysis Software

https://www.trafficware.com/synchro.html

* Guidelines-for-Using-Synchro-9-Including-SimTraffic-9\_(2016), ITS Operations Traffic Management Centre, TORONTO

<https://www.toronto.ca/wp-content/uploads/2017/11/99bc-0_2016-04-28_Guidelines-for-Using-Synchro-9-Including-SimTraffic-9_Final-a.pdf>

* **SUMO: Simulation of Urban Mobility (2015)**. [Institute of Transportation Systems at the German Aerospace Cent](http://www.dlr.de/ts/en/desktopdefault.aspx)er,

<http://sumo.dlr.de/index.html>